Chiral invariant phase space event generator

Deep inelastic lepto-nucleon reactions

M.V. Kossov

CERN, 1211 Geneva, Switzerland

1 Introduction

The CHIPS model in the Geant4 simulation toolkit [1] is used for simulating nuclear fragmentation on a quark-parton level. The simulation for pion capture [2] and for low-energy photo-nuclear reactions [3] reproduces spectra of hadrons and nuclear fragments. Recently, the hadron spectra in muon-nuclear capture have been simulated [4] proving that the model can be applied even at very low energies. The model fits the masses of SU(3) hadrons [5]. Electro- and muon-nuclear reactions are simulated [6] by an equivalent photons method. As most of the equivalent photons have low $Q^2$, it is important to make an accurate approximation for photo-nuclear interactions. However, to simulate neutrino-nuclear interactions, where the low-$Q^2$ transfers are suppressed by the W-boson mass, the model should be generalized to include high $Q^2$.

In the model a temperature of asymptotically free partons is defined by a fundamental constant, $T_c$, a temperature of virtual gluon condensate on the boundary between perturbative and non-perturbative vacuum. This temperature is local. The time-like evolution of the number of soft gluons is roughly defined by a $2T_c$ per parton rule:

$$ s = 4T_c^2 N(s)(N(s) - 1). $$

At high energies and high $Q^2$ one can neglect masses of heavy quark-partons and generalise the CHIPS approach for a space-like evolution. The main problem is the increase of the number of partons in the non-perturbative phase space with increasing $Q^2$. It is assumed that the space-like evolution reveals gluons of the condensate on the boundary between perturbative and non-perturbative vacuum, keeping all partons uniformly distributed over the phase space. The only running constant, defining the number of partons at high $Q^2$, is $\alpha_s(Q^2)$. It is assumed that the number of the revealed gluons is inversely proportional to $\alpha_s(Q^2)$. The fit to structure functions shows that the $Q^2$-dependence of the number of partons is

$$ N(Q^2) = N_0 \frac{1}{2\alpha_s(Q^2)} , $$

where $N_0 = 3$ is the number of valence quark-partons. As the number of partons is increasing, the valence quark fraction of momentum is decreasing with increasing $Q^2$. The number of gluons in eq. (2) should not be confused with the evolution [7] of the number of soft gluons:

$$ N_g(Q^2) = N_g(0) \cdot e^{\frac{4\pi}{\sqrt{2\alpha_s(Q^2)}}} , $$

because the latter describes the interaction splitting of gluons in the cut string, while eq. (2) characterises the appearance of virtual gluons on the boundary between perturbative and non-perturbative vacuum.

The inverse function $\alpha_s(Q^2) = \frac{4\pi}{N_g(Q^2)}$ is shown in fig. 1. The measurements [8] are compared with a generalised LO approximation. The “freezed strong coupling” $\beta_0 = 0.1$, $\ln(1+Q^2/\Lambda^2)$ [9] removes the singularity at $Q^2 = A^2$ ($A^2 = 0.04 \text{GeV}^2$). At $Q^2 = 0$ the gluons vanish.

---

PACS. 02.70.Ss Quantum Monte Carlo method – 12.38.Mh Quark-gluon plasma – 24.85.+p Quarks, gluons, and QCD in nuclei and nuclear processes – 25.30.-c Lepton-induced reactions

DOI 10.1140/epja/i2007-10501-8

2 Structure functions

The hadronic tensor of the non-polarized eN interaction is $W_{\mu\nu} = (-g_{\mu\nu} + \frac{q_{\mu}q_{\nu}}{Q^2}) M \cdot W_1(x, Q^2) + \mathcal{P}_\mu \mathcal{P}_\nu \frac{W_2(x, Q^2)}{M}$, where $p$ is the 4-momentum and $M$ is the mass of the target, $q$ is the 4-momentum of the virtual photon and $\mathcal{P}_\mu = p_\mu - q_\mu \frac{q}{Q^2}$. In the Bjorken limit ($Q^2 \to \infty$) electromagnetic form factors $W_1$ and $W_2$ are connected with the structure functions $F_1$ and $F_2$ of the parton model: $W_2(x, Q^2) \to F_2(x, Q^2)$, $W_1(x, Q^2) \to F_1(x, Q^2) = \frac{x F_2(x, Q^2)}{x M}$, where $\nu$ is the energy of the virtual photon [10]. The $F_1$ and $F_2$ structure functions in the $Q^2 \ll \nu^2$ limit are functions of the transverse and longitudinal cross-sections $\sigma_T$ and $\sigma_L$: $F_1 = \frac{M^2}{x^2 \pi \sigma_T}$ and $F_2 = \frac{Q^2}{4x^2} (\sigma_T + \sigma_L)$, hence

$$ \frac{y d^2 \sigma}{d y d Q^2} = \frac{4 \pi \alpha^2}{Q^4} \left[ (1-y) F_2(x, Q^2) + y^2 x F_1(x, Q^2) \right].$$

Substituting $F_1 = \frac{F_2}{x^2}$ in eq. (4), one can find

$$ \frac{y d^2 \sigma}{d y d Q^2} = \frac{2 \pi \alpha^2}{Q^4} \left[ (2-2y+y^2) F_2(x, Q^2) - y^2 F_1(x, Q^2) \right].$$

The relevant cross-sections including $Q^2 \frac{\alpha^2}{M^2}$ and $\frac{\alpha^2}{Q^2}$ terms, where $M$ is a mass of the target nucleon and $m$ is a mass of the projectile lepton, can be found in [6]. At high $x$ and small $Q^2$, $2xF_1 = F_2 \cdot \frac{1+Q^2}{\nu^2} - F_L$.

In the naive quark-parton model $F_L^{DIS} \equiv 0$ (Callan-Gross condition), because the longitudinal structure function is determined by the transverse momentum distribution of partons $F_L = \frac{4(\gamma^2)}{(Q^2)}$ and the quark-parton model is developed in the $\frac{\gamma^2}{Q^2} \to 0$ limit. Data [11–13] show that $F_L$ is close to zero for $x > 0.01$. The model-dependent analysis of data of the H1 experiment [14] shows that for low $x (0.0001 < x < 0.001$, where $F_2$ is big) $F_L$ is non-zero. The relatively large $F_L(x, Q^2)$ values at low $x$ are usually explained by the interaction of virtual photons with perturbative gluons [9]. At $Q^2 = 0$ the longitudinal structure function $F_L$ is zero by definition, then for the low $x$ at $Q^2 \approx 7\text{GeV}^2$ it rises up to approximately $\frac{1}{4}$ [15], and above $Q^2 > 25\text{GeV}^2$ it decreases inversely proportional to $Q^2$ according to the Bodek-Rock-Yang empirical approximation of the higher twist contributions [16]. The $F_L \neq 0$ problem is closely connected to the $R = \frac{\sigma_T}{\sigma_T^{DIS}} = \frac{F_2}{F_2^{DIS}}$ measurements [17]. Taking into account the $\frac{Q^2}{\nu^2}$ terms mentioned above

$$ R = \frac{F_L}{F_2 \cdot (1 + Q^2/\nu^2)} - F_L.$$

Then eq. (5) can be rewritten in the form

$$ \frac{y d^2 \sigma}{d y d Q^2} = \frac{2 \pi \alpha^2}{Q^4} \left[ 2 - 2y + \frac{y^2}{1 + R} \right] F_2(x, Q^2).$$

The $F_1(x, Q^2)$ structure function can be calculated as the longitudinal component of the scattering on gluons.

The idea to use the phase space distributions of quarks as a starting point for QCD evolution was previously proposed in [18–20]. Recently, the phase space distribution was used [21] for the calculation of the difference between $u$ and $d$ sea-quark distributions in protons. One can consider $N$ partons homogeneously distributed over phase space:

$$ f_N = \frac{dW}{\xi d\xi d(\cos \phi)} = \frac{N(N-1)(N-2)}{2} (1-\xi)^{N-3},$$

where $\xi = \frac{Q}{M}$, $M$ is the mass of the hadronic system and $k$ is the energy of the quark-parton. Equation (8) is normalized by the $\int f_N d\xi = N$ and $\int k f_N d\xi = M$ sum rules.

Interacting with a nucleon, the incident electron radiates a photon with fixed $\nu$ and $Q^2$. To remain massless a quark-parton should have a momentum

$$ k = \frac{Q^2}{2 \cdot (\nu - \cos(\psi) \sqrt{\nu^2 + Q^2})},$$

where $\psi$ is an angle between the quark-parton momentum and the virtual photon momentum. In the $Q^2 \ll \nu^2$ limit the photon momentum is parallel to the electron momentum and $\psi$ becomes an angle with respect to the beam direction ($\theta$). As a result, eq. (9) can be simplified to

$$ 2k \nu (1 - \cos(\theta)) = Q^2.$$

The interaction cross-section for two spin- $\frac{1}{2}$ charges is

$$ \frac{d \sigma}{d Q^2} = -2e^2 \alpha^2 \left( \frac{2}{t^2} + \frac{2}{s^2} + \frac{1}{s^2} \right),$$

where $e$ is the fractional charge of the quark-parton. In the DIS limit ($Q^2 \ll \nu^2$): $-t = Q^2 = 4EE' \sin^2 (\frac{\theta}{2})$ and $s = 4kE \sin^2 (\frac{\theta}{2})$. Taking into account eq. (10), one can find that $s = \frac{Q^2}{\nu}$. Then eq. (11) can be rewritten as

$$ \frac{d \sigma}{d Q^2} = 2e^2 \alpha^2 \left( 2 - 2y + \frac{y^2}{\nu^2} \right) \frac{1}{\nu Q^2} \delta (Q^2 - 2k \nu (1 - \cos(\theta))).$$

Fig. 1. $a_s = 4\pi/\alpha_s$ fitted by a generalized LO formula.