\textbf{f}_0(980)-\text{meson as a K\bar{K} molecule in a phenomenological Lagrangian approach}

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\textbf{Abstract.} We discuss a possible interpretation of the \textit{f}_0(980)-\text{meson as a hadronic molecule} — a bound state of \textit{K} and \textit{\bar{K}} mesons. Using a phenomenological Lagrangian approach we calculate the strong \textit{f}_0(980) \rightarrow \pi\pi and electromagnetic \textit{f}_0(980) \rightarrow \gamma\gamma decays. The compositeness condition provides a self-consistent method to determine the coupling constant between \textit{f}_0 and its constituents, \textit{K} and \textit{\bar{K}}. Form factors governing the decays of the \textit{f}_0(980) are calculated by evaluating the kaon loop integrals. The predicted \textit{f}_0(980) \rightarrow \pi\pi and \textit{f}_0(980) \rightarrow \gamma\gamma decay widths are in good agreement with available data and results of other theoretical approaches.


1 Introduction

The understanding of the structure of scalar mesons with masses around 1 GeV is one of the prominent topics in modern hadronic physics. The study of scalar mesons can, for example, shed light on the problem of the QCD vacuum, \textit{e.g.} to understand the role of gluon configurations and strangeness in the formation of their spectrum. Different interpretations of scalar mesons have been suggested and developed during the last decades [1]. The canonical picture is based on the constituent \textit{q}\bar{q} structure of scalar mesons. In this vein, by analogy with pseudoscalar and vector mesons, one can organize the low-lying scalar mesons, a triplet of \textit{a}_0(980), two doublets of \textit{K}_0^\pm (1430) and two singlets \textit{f}_0(980) and \textit{f}_0(1370), in the form of the \textit{J}^P = 0^+ \text{ nonet} (\text{see, e.g., the discussion in ref. \cite{2}}).

In this paper we focus on the \textit{f}_0(980)-meson. Analyses of the \textit{f}_0(980)-meson as a quarkonium state were performed in several papers (see, \textit{e.g.}, refs. \cite{3-9}). Different scenarios for the admixture of nonstrange and strange \textit{q}\bar{q} components have been developed, which range from a pure or dominant \textit{s}\textit{s} state \cite{3-5} to a dominant \textit{n}\textit{\bar{n}} = (\textit{u}\textit{\bar{u}} + \textit{d}\textit{\bar{d}})/\sqrt{2} configuration \cite{6} with a small \textit{s}\textit{s} mixture of about 10\%. Extensions of this scheme by mixing of quarkonia and glueball components have been analyzed in refs. \cite{7,8}. In \cite{7} it was found that the strong \textit{f}_0(980) \rightarrow \pi\pi decay width is determined by the quarkonium part, while the glueball contribution is small. The dominance of the quarkonium part was also confirmed in ref. \cite{8} in the analysis of the radiative decays of \textit{f}_0(980). In ref. \cite{10} the existence of scalar multiquark states was suggested. Scalar mesons (including the \textit{f}_0(980)-meson) have been assigned to the lightest cryptoelectro \textit{q}\textit{\bar{q}}\textit{q}\textit{\bar{q}}\textit{q} nonet. A further development of the four-quark model for the \textit{f}_0(980) has been done in \cite{11} and recently in \cite{12}. Properties of the \textit{f}_0(980) resulting from the \textit{q}\textit{\bar{q}} and \textit{q}^2\textit{\bar{q}}^2 schemes have been critically analyzed. In \cite{13} the structure of the light scalar nonet including \textit{f}_0(980) was tested using radiative \textit{\phi} decays. The authors of ref. \cite{13} point out the difficulty to distinguish between the \textit{q}\textit{\bar{q}} and the \textit{q}^2\textit{\bar{q}}^2 picture for the light scalar mesons. A possible admixture between \textit{q}\textit{\bar{q}} and \textit{q}^2\textit{\bar{q}}^2 configurations for the low-lying scalar mesons has been considered in ref. \cite{14} using the chiral approach. In ref. \cite{15} the idea of multi-quark states has been put forward to allow for the arrangement of the two quarks and two antiquarks as a bound state of a kaon and an antikaon. Different approaches describing the \textit{f}_0(980) as a hadronic molecule have already been discussed \cite{15-23}. The treatment of the bound-state \textit{K}\textit{\bar{K}} interaction ranges from simple Gaussian \cite{16} and meson exchange \cite{17,18} potentials to chiral perturbation theory (ChPT) \cite{19}. Besides these pure configurations, there exist pictures where mixing is included and the \textit{f}_0(980) incorporates both a \textit{q}\textit{\bar{q}} and a \textit{K}\textit{\bar{K}} or a glueball component \cite{4,24}.
Note that the question on at least the dominant structure of the $f_0(980)$-meson still remains open. Several comprehensive theoretical studies give completely opposing conclusions. E.g., the unitarised meson model of [5] predicts two complete scalar meson nonets, where the $f_0(980)$ is considered as the $s\bar{s}$ ground state. Reference [4] also describes the $f_0(980)$ as a $q\bar{q}$ state but with a large $K\bar{K}$ component due to the proximity to the $K\bar{K}$ threshold. That is the meson spends most of its time in the virtual $K\bar{K}$ state. The analyses of [9, 25] favor the $q\bar{q}$ interpretation of the $f_0(980)$. In particular, ref. [9] predicted two poles close to the $K\bar{K}$ threshold, which suggests that the $f_0(980)$ is a $q\bar{q}$ state with a large $s\bar{s}$ component.

The resonance structure of the $f_0(980)$ was analyzed in [26] by using $J/\psi$ decay data. The $f_0$ was found to be a conventional Breit-Wigner structure. But the data are also compatible with one pole near threshold, which can be identified with a kaon bound state as mentioned in [17]. However, despite this controversial and detailed discussion concerning the $f_0$ structure, the $K\bar{K}$ bound-state configuration seems to be the dominant contribution [21, 22].

In the present paper, the $f_0(980)$ is considered as a pure $K\bar{K}$ molecule in a phenomenological Lagrangian approach. The coupling between the $f_0(980)$-meson and its constituents ($K$ and $\bar{K}$ mesons) is described by the strong-interaction Lagrangian. The corresponding coupling constant is determined by the compositeness condition $Z = 0$ [3, 27], which implies that the renormalization constant of the hadron wave function is set equal to zero. This condition was first applied in order to study the deuteron as a bound state of proton and neutron [27]. Later this method was successfully applied to low-energy hadron phenomenology. It provides the basic equation for the covariant description of mesons and baryons as composite objects of light and heavy constituent quarks, as well as for glueballs which are bound states of gluons (see, e.g., the discussion in refs. [3, 28–32]). Recently, the compositeness condition was also used to study the light scalar mesons $a_0$ and $f_0$ as $K\bar{K}$ molecules [21, 22]. Here, in a first step, we apply our formalism to the study of the strong $f_0(980) \rightarrow \pi\pi$ and electromagnetic $f_0(980) \rightarrow \gamma\gamma$ decays. In particular, previous determinations of the radiative decay width of the $f_0(980)$, for example, when applying the quasi-static approximation, suffer from large uncertainties due to a possible violation of local gauge invariance (for a discussion on this issue, see for example [22]). In the present approach such uncertainties are avoided since we use a fully covariant and gauge-invariant formalism.

In the future we plan to extend the application to the $a_0(980)$ and investigate a possible $f_0(980)$-$a_0(980)$ mixture. Recently, our Lagrangian approach, based on the compositeness condition, was successfully applied to the study of the $D_{s0}(2317)$ and $D_{s1}(2460)$ mesons considered as ($DK$) and ($D^*\bar{K}$) molecules, respectively [33]. In the context of this formalism the strong- electromagnetic- and weak-decay properties of these states have been evaluated.

In the present paper we proceed as follows. First, in sect. 2, we discuss the basic notions of our approach. We derive the phenomenological mesonic Lagrangian including photons for the treatment of the decay properties of the $f_0(980)$-meson as a $K\bar{K}$ bound state. Then, in sect. 3, we discuss the electromagnetic decay $f_0(980) \rightarrow \gamma\gamma$ with the associated diagrams and matrix elements. Special attention will be paid to the proof of electromagnetic gauge invariance. In sect. 4 we turn to the strong decay $f_0 \rightarrow \pi\pi$. Numerical results are discussed in sect. 5, followed by a short summary of our results in sect. 6.

2 Approach

In this section we derive and present the formalism for the study of the $f_0(980)$-meson as a hadronic molecule—a bound state of $K$ and $\bar{K}$ mesons. This means that in our approach the $f_0(980)$ does not decay into a $K\bar{K}$ pair. Our framework is based on an interaction Lagrangian describing the coupling between the $f_0(980)$-meson and its constituents as

$$L_{f_0,KK}(x) = g_{f_0,KK} f_0(x) \int dy \Phi(y^2) K^\dagger(x) K(x),$$

(1)

where $x_\pm = x ± y/2$, $K = (K^+, K^0)$ and $K^\dagger = (K^-, K^0)$ are the doublets of kaons and antikaons, $g_{f_0,KK}$ is the $f_0K\bar{K}$ coupling constant. In particular, the assumed molecular structure of the $f_0(980)$ is in terms of particle content of the form

$$|f_0(980)\rangle = \frac{1}{\sqrt{2}} (|K^+ K^-\rangle + |K^0 K^0\rangle).$$

(2)

The correlation function $\Phi$ in eq. (1) characterizes the finite size of the $f_0(980)$-meson as a $(K\bar{K})$ bound state and depends on the relative Jacobi coordinate $y$ and the center-of-mass (CM) coordinate $x$. The local limit corresponds to the substitution of $\Phi$ by the Dirac delta-function: $\Phi(y^2) \rightarrow \delta^4(y)$. The Fourier transform of the correlation function reads

$$\Phi(y^2) = \int \frac{d^4p}{(2\pi)^4} e^{-ipy} \tilde{\Phi}(-p^2).$$

(3)

Any choice for $\tilde{\Phi}$ is appropriate as long as it falls off sufficiently fast in the ultraviolet region of Euclidean space to render the Feynman diagrams ultraviolet finite. We employ the Gaussian form

$$\tilde{\Phi}(p_E^2) = \exp(-p_E^2/A^2)$$

(4)

for the vertex function, where $p_E$ is the Euclidean Jacobi momentum. Here $A$ is a size constant, which parametrizes the distribution of kaons inside the $f_0$ molecule.

The $f_0K\bar{K}$ coupling constant $g_{f_0,KK}$ is determined by the compositeness condition [3, 27], which implies that the renormalization constant of the hadron wave function is set equal to zero:

$$Z_{f_0} = 1 - \Sigma_{f_0} (M_{f_0}^2) = 0.$$  

(5)

Here $\Sigma_{f_0}(M_{f_0}^2) = g_{f_0,KK}^2 M_{f_0}^2 (M_{f_0}^2)$ is the derivative of the $f_0(980)$-meson mass operator described by the diagram in