Proton-proton scattering without Coulomb force renormalization

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Abstract. We demonstrate numerically that proton-proton (pp) scattering observables can be determined directly by standard short-range methods using a screened pp Coulomb force without renormalization. In the examples the appropriate screening radii are given. We also numerically investigate solutions of the 3-dimensional Lippmann-Schwinger (LS) equation for a screened Coulomb potential alone in the limit of large screening radii and confirm analytically predicted properties for off-shell, half-shell and on-shell Coulomb t-matrices. These 3-dimensional solutions will form a basis for a novel approach to include the pp Coulomb interaction into the AN Faddeev framework.


1 Introduction

The action of the Coulomb force in pp scattering can be rigorously treated using the Vincent-Phatak method [1]. We propose an alternative manner using a screened Coulomb force, despite the well-known fact that the screening limit does not exist. Namely the pp on-shell scattering amplitude acquires an oscillating phase factor if the screening radius goes to infinity [2–5]. This phase factor is known and can be removed, a step known in that context under the name renormalization. However, as we shall show, if one is interested in the pp observables (not in the phase shifts) where that phase factor drops out, all scattering observables can be obtained in the standard framework of short-range interactions. This will be demonstrated in sect. 2 for suitably chosen screening radii.

In view of a forthcoming paper [6] related to the pd system, we further investigate in sect. 3 the properties of the screened 3-dimensional Coulomb t-matrix \( \langle \vec{p}'' | t^R_2(E) | \vec{p} \rangle \). This t-matrix is a solution of the 3-dimensional 2-body LS equation driven by the screened Coulomb potential. Namely, to catch the full action of the Coulomb force in the pd system a partial-wave–truncated pp t-matrix is insufficient and the complete 3-dimensional Coulomb t-matrix has to be used. Analytical properties of that screened Coulomb t-matrix, off-the-energy-shell, half-shell and on-shell have been studied in the past [2–4, 7, 8]. These investigations, however, mostly rely on insights gained for fixed partial-wave states. The mathematical rigor in the summation of the partial-wave sum to infinity leaves room for improvement. Therefore we felt that a numerical study is justified to verify the statements given there: the screening limit of \( \langle \vec{p}'' | t^R_2(E) | \vec{p} \rangle \) exists for \( R^2 \neq E \neq \frac{p^2}{m} \) and coincides with the unscreened pure Coulomb force expression, which is known analytically [2, 9] and references therein; that screening limit exhibits a discontinuity if \( p \) approaches \( \sqrt{m p E} \), \( E > 0 \) from above or below; the screening limit of the on-shell t-matrix \( \langle \vec{p} | t^R(E = \frac{p^2}{m}) | \vec{p} \rangle \) approaches the analytically known unscreened Coulomb on-shell t-matrix up to a given infinitely oscillating phase factor. Here we want to numerically investigate at which \( R \)-values these limits are reached with adequate accuracy. We conclude in sect. 4.

2 The on-shell pp t-matrix with screened Coulomb potential and the pp observables

Let \( V^R_c \) be the screened Coulomb potential between 2 protons normalised such that \( V^R_c \) turns into the pure pp Coulomb potential for \( R \), the screening radius, going to infinity. Together with the strong interaction \( V \) this determines the 2-body pp t-matrix via the LS equation

\[
t = V + V^R_c + (V + V^R_c)G_0t,
\]

where \( G_0 \) is the free propagator. That equation is solved at the pp c.m. energy \( E = \frac{p^2}{m} \) projected on a set of partial-wave basis states \( |p(l)s jm; tm_l) \), with \( p, l, s, j \) and \( m \)
the relative momentum, orbital angular momentum, total spin, total angular momentum and its magnetic quantum number.

The total isospin quantum numbers for two protons are \( t = 1 \) and \( m_t = -1 \). This leads to the on-the-energy-shell \( t \)-matrix element

\[
(p' l' s') m' |t(p l s)jm\rangle = \delta_{s's} \delta_{j'j} \delta_{m'm} \langle \vec{t}_{li}(p,p),
\]

where the Pauli principle dictates \((-)^{j+s} = 1\) and we took \( s \) to be conserved.

The full 3-dimensional antisymmetrized on-shell \( t \)-matrix is given as

\[
\langle \vec{t} m'_1 m'_2 | t(1 - P_{12}) | \vec{p} m_1 m_2 \rangle,
\]

where \( m_i = \{p' = \vec{p}, p = \vec{p}'\} \) the initial and final relative momenta.

The standard partial-wave decomposition leads to

\[
\langle \vec{t} m'_1 m'_2 | t(1 - P_{12}) | \vec{p} m_1 m_2 \rangle = \sum_s \left[ \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \right] \langle \vec{s} s' m'_1 m'_2 | t(1 - P_{12}) | \vec{s} s m_1 m_2 \rangle
\]

\[
\times \sum_{j=0}^{\infty} \sum_{j'=-j}^{j} \sum_{j''=-j}^{j} (l' s_j, m'_1, m'_2, m) Y_{l'm'_1}(\vec{p}) (1 + (-)^{l+s}) \sum_{m_l} (l s_j, m_l m_s, m) Y_{l'm_l}(\vec{p}).
\]

The strong force can be neglected beyond a certain \( j_{\text{max}} \) and there only the screened Coulomb \( t \)-matrix \( t^R_{12} \) is present, which is diagonal in \( l \) and independent of \( s \) and \( j \). In a well-known manner one adds and subtracts a finite sum up to \( j_{\text{max}} \) with \( t^R_{12} \) only and this completes the infinite sum over \( j \) containing only \( t^R_{12} \). That infinite sum is identical to the 3-dimensional antisymmetric screened Coulomb \( t \)-matrix. Thus \( (4) \) turns into

\[
\langle \vec{p}' m'_1 m'_2 | t(1 - P_{12}) | \vec{p} m_1 m_2 \rangle = \delta_{m'_1 m} \delta_{m_2 m_2}
\]

\[
\times \sum_s \left[ \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \right] \langle \vec{s} s' m'_1 m'_2 | t(1 - P_{12}) | \vec{s} s m_1 m_2 \rangle
\]

\[
\times \sum_{j=0}^{\infty} \sum_{j'=-j}^{j} \sum_{j''=-j}^{j} (l' s_j, m'_1, m'_2, m) Y_{l'm'_1}(\vec{p}) (1 + (-)^{l+s}) \sum_{m_l} (l s_j, m_l m_s, m) Y_{l'm_l}(\vec{p}).
\]

Now as is well known [3,4] the limit of that expression does not exist for \( R \rightarrow \infty \). In that limit each term in \( (5) \) acquires the same infinitely oscillating factor \( e^{2i\Phi(p)} \), where \( \Phi(p) \) is given below. If one is interested in scattering phase shifts it is unavoidable to keep track of this oscillating factor which in that context runs under the name renormalization [5]. However, if one is interested in

the \( pp \) observables, the cross-section and all sorts of spin observables (note \( \Phi(p) \) is independent of spin magnetic quantum numbers), where the on-shell \( t \)-matrix appears together with its complex conjugate, the oscillating factor drops out. In that case one does not even has to know the analytical form of \( \Phi(p) \). It is sufficient to know that the limit of large screening radius generates just a phase factor.

This is the main message of this section: the \( pp \) observables based on the strong and the screened Coulomb force can be calculated without renormalization using standard short-range methods. Though not explicitly stated in [3, 4] this insight is in the spirit of these authors. It remains to establish the values of the parameter \( R \) at which the observables get independent of \( R \).