Gaussian Effective Potential and superconductivity

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Abstract. The Gaussian Effective Potential in a fixed transverse unitarity gauge is studied for the static three-dimensional U(1) scalar electrodynamics (Ginzburg-Landau phenomenological theory of superconductivity). In the broken-symmetry phase the mass of the electromagnetic field (inverse penetration depth) and the mass of the scalar field (inverse correlation length) are both determined by solution of the coupled variational equations. At variance with previous calculations, the choice of a fixed unitarity gauge prevents from the occurrence of any unphysical degree of freedom. The theory provides a nice interpolation of the experimental data when approaching the critical region, where the standard mean-field method is doomed to failure.

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1 Introduction

Since the discovery of high $T_c$ superconductors several unconventional models have been proposed in order to describe the unusual properties of cuprates. In particular, the strong electron-electron coupling characterizing such materials requires new theoretical methods beyond the standard mean field approach. On the other hand the general phenomenology is still well described by the standard Anderson-Higgs mechanism: the supercurrent is carried by pairs of charged fermions whose non-vanishing expectation value breaks the gauge symmetry, thus endowing the gauge bosons with a mass. Thus the standard Ginzburg-Landau (GL) effective Lagrangian still provides the best framework for a general description of the high-$T_c$ cuprates phenomenology. Moreover, as the GL action can be seen as a power expansion of the exact action around the critical point, the GL action must be recovered by any microscopic theory at least around the transition. Thus, regardless of the nature of the pairing mechanism, the GL action is a sound starting point for a general description of the high-$T_c$ materials. Of course we cannot trust the mean-field approach to the GL effective theory, and we expect that many unconventional properties are connected with the breaking down of the simple mean field picture. Quite recently, the GL model has been extensively studied both theoretically [1,2] and numerically [3] in order to clarify the universality class of the superconducting transition and the role of the critical fluctuations in the high-$T_c$ cuprates, as well as the order of the transition itself [4].

Actually the high-$T_c$ cuprate superconductors are characterized by a very small correlation length $\xi$ which allows the experimentalists to get closer to the critical point where the thermal fluctuations cannot be neglected and the mean field approximation is doomed to fail [5]. As far as we know there is no full evidence that the universal critical behaviour has been reached in any real sample [6], but it is out of doubt that an intermediate range of temperature is now accessible, where thermal fluctuations are not negligible even if the sample is still out of the truly critical regime. Thus, in order to describe some unconventional properties of the high-$T_c$ superconductors, we need to incorporate the role of thermal fluctuations, but unfortunately we cannot rely on the standard renormalization group methods which would only describe the universal limiting behaviour that could not be observed yet in any real sample. We need some kind of interpolation scheme for the non-universal regime where the behaviour depends on the physical parameters of the sample, and we would prefer a non-perturbative method in order to deal with any strong coupling.

In this paper we study the Gaussian fluctuations by means of a variational method, the Gaussian Effective Potential (GEP), which has been discussed by several authors as a tool for describing the breaking of symmetry in a simple scalar theory [7,8]. As a toy model for electro-weak

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interactions, the scalar electrodynamics in four dimensions has been studied by Ibañez-Meier et al. [9] who computed the GEP by use of a general covariant gauge. However their method gives rise to an unphysical, and undesirable, degree of freedom.

We compute the GEP for the U(1) scalar electrodynamics in three space dimensions where it represents the standard static GL effective model of superconductivity. In order to make evident the physical content of the theory, thus avoiding the presence of unphysical degrees of freedom, we work in unitarity gauge. This has been shown to be formally equivalent to a full gauge-invariant method once all the gauge degrees of freedom have been integrated out [10]. The variational method provides a way to evaluate both the correlation length $\xi$ and the penetration depth $\ell$ as a solution of coupled equations. The GL parameter $\kappa_{\text{GL}} = \ell/\xi$ is found to be strongly temperature dependent in contrast to the simple mean-field description [11].

On the other hand the model predictions are in perfect agreement with some recent experimental data [12], which can be nicely interpolated by our variational calculation. The comparison with the experimental data is of special importance as it provides a test for the GEP variational method itself. The predictions of the method in 3+1 dimensions, in the context of electro-weak interactions, have been discussed by several authors [7,9,13,14], but no real comparison with experimental data will be achievable until the detection of the Higgs boson. Thus we compute the GEP for the U(1) scalar electrodynamics in three space dimensions where it represents the standard static GL action [15] as a solution of coupled equations. The GL param-

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2 The GL action in unitarity gauge

Let us consider the standard static GL action [15]

$$S = \int \! d^3x \left[ \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} (D_\mu \phi)^* (D^\mu \phi) + \frac{1}{2} m_\phi^2 \phi^* \phi + \lambda_B (\phi^* \phi)^2 \right].$$  \hspace{1cm} (1)

Here $\phi$ is a complex (charged) scalar field, whose covariant derivative is defined as

$$D_\mu \phi = \partial_\mu + i e_B A_\mu$$  \hspace{1cm} (2)

and $\mu, \nu = 1, 2, 3$ run over the three space dimensions. The magnetic field components $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ satisfy

$$\frac{1}{2} F_{\mu\nu} F^{\mu\nu} = |\nabla \times A|^2$$  \hspace{1cm} (3)

and the partition function is defined by the functional integral

$$Z = \int \! D[\phi, \phi^*, A_\mu] e^{-S}. \hspace{1cm} (4)$$

We may assume a transverse gauge $\nabla \cdot A = 0$, and then switch to unitarity gauge in order to make $\phi$ real. Let us define two real fields $\rho$ and $\gamma$ according to $\phi = \rho e^{i\gamma}$. The unitarity gauge is recovered by the gauge transformation

$$A \rightarrow A - \frac{1}{e_B} \nabla \gamma(x). \hspace{1cm} (5)$$

and the original transverse vector field $A_\perp$ acquires a longitudinal component $A_L$ proportional to $\nabla \gamma$. Thus the original measure in equation (4) becomes

$$\int \! D[\rho, \phi^*, A_L] = \int \! D[\rho] D[D[A_L] \rightarrow \text{const.} \times \int \! \rho D[\rho] D[A_L] D[A_\perp]. \hspace{1cm} (6)$$

In unitarity gauge the action, equation (1), now reads

$$S = \int \! d^3x \left[ \frac{1}{2} (\nabla \rho)^2 + \frac{1}{2} m_\rho^2 \rho^2 + \lambda_B \rho^4 + \frac{1}{2} \epsilon_B^2 \rho^2 (A_L^2 + A_\perp^2) + \frac{1}{2} (\nabla \times A_\perp)^2 \right] \hspace{1cm} (7)$$

and the longitudinal field $A_L$ may be integrated out exactly yielding a constant factor and an extra $1/\rho$ factor for the measure (6). Finally, dropping the constant factors, the partition function may be written as

$$Z = \int \! D[\rho, A_\perp] \exp \left\{ - \int \! d^3x \left[ \frac{1}{2} (\nabla \rho)^2 + \frac{1}{2} m_\rho^2 \rho^2 + \lambda_B \rho^4 + \frac{1}{2} \epsilon_B^2 \rho^2 A_L^2 + \frac{1}{2} (\nabla \times A_\perp)^2 \right] \right\}. \hspace{1cm} (8)$$

We may enforce the transversal condition on the vector field by a gauge fixing term in the action and, restoring $\rho = \phi$, the action reads

$$S = \int \! d^3x \left[ \frac{1}{2} (\nabla \phi)^2 + \frac{1}{2} m_\phi^2 \phi^2 + \lambda_B \phi^4 + \frac{1}{2} \epsilon_B^2 \phi^2 A^2 + \frac{1}{2} (\nabla \times A)^2 + \frac{1}{2} (\nabla \cdot A)^2 \right]. \hspace{1cm} (9)$$

The partition function is now expressed as a functional integral over the real scalar field $\phi$ and the generic three-dimensional vector field $A$, with the extra prescription that the parameter $\epsilon$ is set to zero at the end of the calculation. Inserting a source term we may write

$$Z[j] = \int \! D[\rho, A_\mu] \exp \left\{ - S + \int \! d^3x j \phi \right\}$$  \hspace{1cm} (10)

with $S$ given by equation (9). The free energy (effective potential) follows by the Legendre transformation

$$F[\varphi] = - \ln Z + \int \! d^3x j \varphi$$  \hspace{1cm} (11)

where $\varphi$ is the average value of $\phi$ in presence of the source $j$. The superconducting phase is characterized by an absolute minimum of $F$ for $\varphi \neq 0$. 