Current and efficiency enhancement in Brownian motors driven by non Gaussian noises

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Abstract. We study Brownian motors driven by colored non Gaussian noises, both in the overdamped regime and in the case with inertia, and analyze how the departure of the noise distribution from Gaussian behavior can affect its behavior. We analyze the problem from two alternative points of view: one oriented mainly to possible technological applications and the other more inspired in natural systems. In both cases we find an enhancement of current and efficiency due to the non-Gaussian character of the noise. We also discuss the possibility of observing an enhancement of the mass separation capability of the system when non-Gaussian noises are considered.

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1 Introduction

The study of noise induced transport by “ratchets” has attracted in recent years the attention of an increasing number of researchers due to the biological interest and also to its potential technological applications [1,2]. Since the pioneering works, besides the built-in ratchet-like bias and correlated fluctuations (see for instance [3]), different aspects have been studied, such as tilting [4,5] and pulsating [6] potentials, velocity inversions [4,7], etc. There are some relevant reviews [8,9] where the biological and/or technological motivation for the study of ratchets can be found.

Recent studies on the role of non Gaussian noises on some noise-induced phenomena like stochastic resonance, resonant trapping, and noise-induced transitions [10–15] have shown the possibility of strong effects on the system’s response. For instance, enhancement of the signal-to-noise ratio in stochastic resonance, enhancement of the trapping current in resonant trapping, or shifts in the transition line for noise-induced transitions. These results motivate the interest in analyzing the effect of non Gaussian noises on the behavior of Brownian motors. Here we analyze the effect of a particular class of colored non Gaussian noise on the transport properties of Brownian motors. Such a noise source is based on the nonextensive statistics [16,17] with a probability distribution that depends on $q$, a parameter indicating the departure from Gaussian behavior: for $q = 1$ we have a Gaussian distribution, and different non Gaussian distributions for $q > 1$ or $q < 1$.

Some of the motivations for studying the effect of non Gaussian noises are, in addition to its intrinsic interest within the realm of noise induced phenomena, the existence of experimental data indicating that for several biological problems fluctuations have a non Gaussian character. Examples are current measurements through voltage-sensitive ion channels in a cell membrane or experiments on the sensory system of rat skin [18]. Also, recent detailed studies on the source of fluctuations in different biological systems [19] clearly show that, in such a context, noise sources are in general non Gaussian. Even though the previous arguments refer to biological aspects that are not directly related to ratchets, they strongly induce to think about the possible relevance of considering non Gaussian noises in those biological situations where the ratchet transport mechanism can play a role. In addition, from the point of view of technological applications, the finding of new conditions that may lead to an enhancement of the efficiency of the devices is always desirable. It is worth here remarking that there are some previous studies of non-Gaussian noise with similar tails [20]. For instance, those authors have analyzed aspects of the periodic attractors emerging from a saddle-node bifurcation plus noise that show chaos, or escape rates in noisy maps.

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We show here that, as a consequence of the non Gaussian character of the driving noise and from two alternative points of view, we can find a kind of enhancement of the system’s response. The first – direct – point of view, following the line of previous works [10,12–15], takes as free parameters those that could be controlled in the case of technological applications. In this case we find a remarkable increase of the current together with an enhancement of the motor efficiency when non-Gaussian noises are considered (showing an optimum for a given degree of departure from the $q = 1$ Gaussian behavior). Moreover, when inertia is taken into account it is found that, again when departing from the Gaussian case, there seems to be a remarkable increment in the mass separation capability of these devices. The second point of view [11] is the more natural one when thinking of biological systems, as it considers the non-Gaussian noise as a primary source. In this case we also find an enhancement of the current and efficiency due to departure from Gaussian behavior, which occurs for relative low values of noise intensity. And the possibility of an inversion of current due only to changes on the non gaussianity of the noise.

Our analysis will include numerical simulations and analytical results coming from an adiabatic approximation valid for high correlation time of the forcing. We begin presenting the general framework within which we will work, and the nature of the non Gaussian noise. We continue discussing the first of the two points of view, and the results showing the enhancement we can find within it. After that we discuss the second point of view where we compare Gaussian and non Gaussian behaviors but adopting a constant width criterion, and discuss the results. Finally we draw some general conclusions.

2 Framework

We begin considering the general system
\[ m \frac{d^2 x}{dt^2} = -\gamma \frac{dx}{dt} - V'(x) - F + \xi(t) + \eta(t), \]
where $m$ is the mass of the particle, $\gamma$ the friction constant, $V(x)$ the ratchet potential, $F$ is a constant “load” force, and $\xi(t)$ the thermal noise satisfying $\langle \xi(t)\xi'(t') \rangle = 2\gamma T \delta(t-t')$. Finally, $\eta(t)$ is the time correlated forcing (with zero mean) that allows the rectification of the motion, keeping the system out of thermal equilibrium even for $F = 0$. For this type of ratchet model several different kinds of time correlated forcing have been considered in the literature [8,9]. In almost all studies authors have used Gaussian noises. The few exceptions which considered non Gaussian processes correspond mainly to the case of dichotomic noises [2,4,9].

The main characteristic introduced by the non Gaussian form of the forcing we consider here, is the appearance of arbitrary strong “kicks” with relatively high probability when compared, for example, with the Gaussian Ornstein–Uhlenbeck (OU) noise and, of course, with the dichotomic non Gaussian process.

2.1 Noise source

We will consider the dynamics of $\eta(t)$ as described by the following Langevin equation [10]
\[ \frac{d\eta}{dt} = -\frac{1}{\tau} \frac{d}{dt} V_q(\eta) + \frac{1}{\tau} \zeta(t), \]
with $\langle \zeta(t) \rangle = 0$ and $\langle \zeta(t)\zeta'(t') \rangle = 2D \delta(t-t')$, and
\[ V_q(\eta) = \frac{D}{\tau(q-1)} \ln \left[ 1 + \frac{\tau}{D} (q-1) \eta^2 \right] . \]

Previous studies of such processes in connection with stochastic resonance problems [10,11] and dynamical trapping [13], have shown that the non Gaussian behavior of the noise leads to remarkable effects. For $q = 1$, the process $\eta$ coincides with the OU one (with a correlation time equal to $\tau$), while for $q \neq 1$ it is a non Gaussian process. As shown in [10], for $q < 1$ the stationary probability distribution has a bounded support, with a cut-off at $|\eta| = \omega \equiv [(1-q)\tau/(2D)]^{-\frac{1}{2}}$, with a form given by
\[ P_q(\eta) = \frac{1}{Z_q} \left[ 1 - \left( \frac{\eta}{\omega} \right)^2 \right]^{-\frac{1}{2}}, \]
for $|\eta| < \omega$ and zero for $|\eta| > \omega$ ($Z_q$ is a normalization constant). Within the range $1 < q < 3$, the probability distribution is given by
\[ P_q(\eta) = \frac{1}{Z_q} \left[ 1 + \frac{\tau(q-1)\eta^2}{2D} \right]^{-\frac{1}{2}} \]
for $-\infty < \eta < \infty$, and decays as a power law (slower than a Gaussian distribution). Finally, for $q > 3$, this distribution cannot be normalized.

Hence, we see that keeping $D$ constant, the width or dispersion of the distribution increases with $q$. This means that, the higher the $q$, the stronger the “kicks” that the particle will receive. Figure 1 depicts the typical form of this distribution for $q$ smaller, equal and larger than 1.

In [10] it was shown that the second moment of the distribution, which we will interpret as the “intensity” of the non Gaussian noise, is given by
\[ \langle \eta^2 \rangle = \frac{2D}{\tau(5-3q)}, \]
which diverges for $q \geq 5/3$. For the correlation time $\tau_{\eta}$ of the process $\eta(t)$, defined in detail in [10] it is not possible to find an analytical expression. However, it is known [10] that for $q \rightarrow 5/3$ it diverges as $\sim (5-3q)^{-1}$. In our analysis, we will consider values of $q$ in the range $0.5 < q < 5/3 \approx 1.66$. For this interval we have studied numerically the dependence of $\tau_{\eta}$ on $q$, and we have found the following analytical approximation
\[ \tau_{\eta} \approx 2 \left[ 1 + 4(q-1)^2 \right] \frac{\tau}{(5-3q)} . \]