Electronic polarizability of superconductors and inertial mass of a moving vortex

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Abstract. The problem of a vortex electromagnetic mass in a superconductor is considered accounting for the self-interaction effect conditioned by the coupling of the moving vortex to the excited fluctuations of the superfluid density. The obtained polaron-type mass exceeds the earlier obtained electromagnetic mass in view of the large value of the light speed relation to the Fermi velocity and can dominate over the vortex core mass.

PACS. 74.20.De Phenomenological theories (two-fluid, Ginzburg-Landau, etc.) – 74.25.Qt Vortex lattices, flux pinning, flux creep

Motion of Abrikosov vortices has a crucial impact on basic characteristics of superconductors, therefore the different regimes of vortex lattice dynamics has been studied for conventional and high-temperature superconductors. In the underdamped regime, when the equation of motion for the vortex cannot be reduced to the balance condition for external and damping forces, an important dynamical property is the inertial mass of the moving vortex. The discussion of effects arising due to the vortex mass can be found e.g. in references \cite{1,2}. The inertial mass is an effective parameter that characterizes the increase of the internal energy of the superfluid when the vortex moves with a velocity $v$, owing to origination of the kinetic energy $F_{\text{kin}} = M v^2/2$. Various sources of a vortex kinetic energy in superconductors were proposed, such as the energy of the induced electromagnetic fields \cite{2–6}, the core energy \cite{2,3,5–9}, the energy of the hydrodynamic backflow \cite{10}, and the crystal lattice deformation energy \cite{11–13}.

Generation of the electromagnetic mass is the result of the polarization of the charged superfluid around the vortex core. In the region far from the core the modulus of the superconducting order parameter is roughly constant and the vortex motion causes only fluctuations of the order parameter phase, which implies excitation of the density fluctuations and the local polarization of the superfluid. In neutral compressible superfluids the density oscillations stimulated by a vortex motion create a significant “compressibility mass” \cite{10,14,15}, exceeding the core contribution. Meanwhile, in the charged liquids the density fluctuations are accompanied by the creation of electric fields which are strongly screened due to the Coulomb interaction. This screening reduces the energy of the density fluctuations. The electromagnetic mass of a vortex was first studied by Suhl \cite{3} at the temperature $T = 0$ under the assumption of the perfect charge screening. In this work the induced electric field has been determined from the local charge neutrality condition, and the electric field energy was designated as the vortex kinetic energy. More accurate estimate of the electromagnetic mass at zero temperature has been made by Duan and Leggett \cite{5} and Duan \cite{6} with account of compressibility of the superfluid. In these works the local charge density and the electric field were computed by the self-consistent way using Maxwell’s equations, while the vortex inertia was conditioned both by the electric field and the charge density oscillation energies. Nevertheless, the electromagnetic mass obtained by Suhl \cite{3} coincides with the results of the papers \cite{5,6} due to the fact that the charge screening length in superconductors is much smaller than the correlation length $\xi$. The electromagnetic mass found in the above-mentioned works is exceeded by the vortex core mass owing to the smallness of the Fermi velocity $v_F$ with respect to the light speed $c: M_{el}/M_{\text{core}} \propto (\lambda_L/\xi)^2 (v_F/c)^2$, where $\lambda_L$ is the London penetration length.

Although in the papers \cite{5,6} the charged superfluid compressibility was taken into account which permitted the correct description of the excited density oscillations, the role of vortex interaction with these excitations must be clarified for the proper determination of the vortex electromagnetic mass. A model of the vortex coupling to the low lying excitations of a neutral superfluid was proposed by Niu, Ao and Thouless \cite{16}. We show in this paper that in superconductors the interaction of a vortex with dynamical polarization of the background permits an
obvious description in the framework of the classical electrodynamics. This interaction generates a large electromagnetic mass, that can exceed the vortex core mass. The problem is discussed for the temperature region near absolute zero, when the concentration of normal electrons and the dissipation outside the vortex core vanish.

A vortex in a superfluid is the topological object that can be described by the phase \( \chi \) of the order parameter \( \psi = \Delta \exp(\int f \nabla \chi \, dl) \). If \( L \) is a closed contour encircling the point \( \mathbf{r} = \mathbf{0} \) in the \((x,y)\) plane, which is the coordinate of the singularity line directed along the \( z \)-axis of the coordinate system, then the single-valuedness of the order parameter is expressed by the equation \( \oint \nabla \chi \, dl = 2\pi \), allowing one to identify the phase around the static vortex with the azimuthal angle \( \theta = \arctg(y/x) \). In the case when the vortex moves uniformly with a small velocity \( \mathbf{v} \) (when the adiabatic approximation for the phase is valid [15]), the phase is determined as \( \chi(\mathbf{r}, t) = \theta(\mathbf{r} - \mathbf{vt}) \). The space and time derivatives of this phase

\[
\nabla \chi = \frac{1}{\mathbf{r}}, \dot{\chi} = -\mathbf{v} \nabla \chi \tag{1}
\]

are the sources of the fields and currents around the moving vortex.

Induction of magnetic and electric fields by a singularity in superconductors results directly from the gauge invariance: the phase derivatives (1) enter into the energy functional in the proper combinations with the vector potential \( \mathbf{A} \) and the scalar potential \( \varphi \). In the region far from the core, where the order parameter modulus is constant, the energy functional[3] can be written as follows:

\[
F = F_0 + \int d\mathbf{r} \left\{ \gamma_0 \left( \dot{\chi} + \frac{2e}{\hbar} \varphi \right)^2 + \gamma \left( \nabla \chi - \frac{2e}{\hbar c} \mathbf{A} \right)^2 + \frac{\mathbf{B}^2 + \mathbf{E}^2}{8\pi} \right\}. \tag{2}
\]

Here \( F_0 \) is the energy of the homogeneous superconductor, i.e. the computing origin of the vortex energy; \( \mathbf{B} \) and \( \mathbf{E} \) are the induced magnetic and electric fields respectively. The integration goes over the two-dimensional radius-vector \( \mathbf{r} \), as the vortex unit length mass should be estimated. The functional (2) describes only the phase fluctuations, therefore the parameters \( \gamma_0 \) and \( \gamma \) are dependent on the order parameter amplitude and determine the charge density and the current. The coefficient \( \gamma_0 \) can be expressed through the charge screening distance \( \lambda_{scr} \), and \( \gamma \) - through the London penetration length \( \lambda_L \):

\[
\gamma_0 = \frac{1}{8\pi c} \left( \frac{\phi_0}{2\pi \lambda_{scr}} \right)^2, \quad \gamma = \frac{1}{8\pi} \left( \frac{\phi_0}{2\pi \lambda_L} \right)^2, \tag{3a}
\]

where \( \phi_0 = \pi \hbar c/e \) is the flux quantum. The ratio of these parameters gives the square of the characteristic velocity

\[
s^2 = \gamma/\gamma_0. \tag{3b}
\]

In the energy functional used by Suhl [3] and in the further works [2, 5, 6] this velocity was assumed to be equal \( s = v_F/\sqrt{3} \), then \( \lambda_{scr} \) represents the Fermi-Thomas screening length. Here, analyzing the problem in the framework of the phenomenological theory, we will not involve as yet the microscopic values of the parameters \( \gamma_0 \) and \( \gamma \), preferring to use the phenomenological parameters \( s \) and \( \lambda_L \) determined by the equations (3a) and (3b).

The external magnetic field that has penetrated into the superconductor and created the topological defect can be described by a vector potential defined as \( \mathbf{A}^{ext} = -\left(\phi_0/2\pi\right) \nabla \chi \). The induced magnetic field and the vortex currents which screen the external field are described by the potential \( \mathbf{A} \equiv \mathbf{A}^{ind} \), so that the magnetic field \( \mathbf{B} \) in (2) is \( \mathbf{B} = \nabla \times \mathbf{A}^{ind} \). In a similar manner, we can define the external electric field \( \varphi^{ext} = \left(\phi_0/2\pi c\right) \chi \) which is screened by the induced charge density and creates the scalar potential \( \varphi^{ind} \). As the slow motion of the vortex keeps the phase gradient (1) purely transverse, the induced vector-potential \( \mathbf{A}^{ind} \) also have no longitudinal component (due to the gauge coupling of the order parameter to the electromagnetic field) and its time derivative determines the transverse field \(-\partial_t \mathbf{A}^{ind}/c\). The electric field in (2) represents the sum of the transverse and longitudinal components:

\[
\mathbf{E} = -\partial_t \mathbf{A}^{ind}/c - \nabla \varphi^{ind}. \tag{4}
\]

Let us now analyze in detail the origin of the longitudinal electric field around the moving vortex. The total electric field represents the sum of the induced scalar potential gradient and of the electric field generated by the moving magnetic field \( \mathbf{B} \):

\[
\mathbf{E}^{total} = -\varphi^{ind} - \frac{1}{c} (\mathbf{v} \times \mathbf{B}). \tag{5}
\]

This expression can be obtained from the hydrodynamic equation for the charged superfluid flow in the presence of a magnetic field [10]. Essentially, the second term in the right-hand side of this expression contains the longitudinal component \( [\mathbf{v} \times \mathbf{B}]^t/c \). This component can be revealed using \( \mathbf{B} = \nabla \times \mathbf{A}^{ind} \) and the vector transformation \( [\mathbf{v} \times \mathbf{B}] = - (\nabla \varphi^{ind}) \mathbf{A}^{ind} + \nabla \times (\mathbf{v} \mathbf{A}^{ind}) \). Here the first term is equal to \( \partial_t \mathbf{A}^{ind} \) and is responsible for the transverse electric field induction, while the second term creates a longitudinal field along with the scalar potential gradient. Thus the total electric field is equal to

\[
\mathbf{E}^{total} = -\frac{1}{c} \partial_t \mathbf{A}^{ind} - \nabla \left( \varphi^{ind} + \frac{1}{c} \mathbf{v} \mathbf{A}^{ind} \right). \tag{6}
\]

The obtained longitudinal electric field is due to the transformation of the scalar potential in the laboratory frame (connected with the superconductor) which is the reference frame for the magnetic field motion:

\[
\varphi = \varphi^{ind} + \frac{1}{c} \mathbf{v} \mathbf{A}^{ind} \tag{7}
\]

The scalar potential (4) is just the one that must compose the gauge-invariant combination with the phase time-derivative that enters into the functional (2). Usually in this functional the scalar potential is assumed to be equal to \( \varphi^{ind} \), omitting the second component in (4). Meanwhile, the account of this component of the scalar potential allows one to obtain the longitudinal current and