Associated relaxation time and intensity correlation function of a bistable system driven by cross-correlation additive and multiplicative coloured noise sources

Ping Zhu

Department of Physics, Simao Teacher’s College, Simao 665000, China

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Abstract The associated relaxation time and the intensity correlation function of a bistable system driven by an additive and a multiplicative coloured noise with coloured cross-correlation are investigated. Using the Novikov theorem and the projection operator method, the analytic expressions of the stationary probability distribution \(P_{st}(x)\), the relaxation time \(T_c\), and the normalized correlation function \(C(s)\) of the system are obtained. The effects of the noise intensity, the cross-correlation strength \(\lambda\) and the cross-correlation time \(\tau\) are discussed. By numerical computation, it is found that the cross-correlation strength \(|\lambda|\) and the quantum noise intensity \(D\) decrease the relaxation of the system from unstable points. The cross-correlation time \(\tau\) delays relaxation of the system from unstable points. The cross-correlation strength \(\lambda\) and the cross-correlation time \(\tau\) can alter the effects of the pump noise intensity \(Q\). Thus, the relaxation time \(T_c\) is a stochastic resonant phenomenon, and distribution curves exhibit a single-maximum structure.

PACS. 05.40.-a Fluctuation phenomena – 02.50.-r Probability theory, stochastic processes – 05.10.Gg Stochastic analysis methods

1 Introduction

A bistable system with noise is a typical and important problem in statistical mechanics. It is related to many practical problems, including quenching phenomena [1–3], bistable optical systems [4, 5], stochastic resonant phenomena [6–9], etc. In most previous work, noise sources are usually treated as uncorrelated random variables, since it is usually assumed that they have different origins. However, in some practical cases, sources of noise may have a common origin, and hence can be correlated [11, 12]. There are other situations where strong external noise can engender changes in the internal structure of a system so that the internal noise and the external noise should be independent [12–15]. Bistable systems with correlation noise terms are the subject of other studies [15–27]. Hanggi et al. first investigated colour effects in the activation rate of a bistable system [14]. Marchi et al. studied the resonant activation for a bistable system driven by an additive and a multiplicative noise [17]. Using the Novikov theorem and the steady-state value of the deterministic theory, Jia and Li analyzed the steady-state properties of the bistable kinetic model with cross-correlation additive and multiplicative white noise [28].

The associated relaxation time and the intensity correlation function are important physical quantities to characterize the statistical behaviour of a stochastic process, and hence are usually used to describe the fluctuation behaviour of a nonlinear system [29]. Research into the problem has shown that the associated relaxation time and the intensity correlation function for nonlinear stochastic systems are important physical features [30–33]. Applying the projection operator method, Xie and Mei investigated dynamical properties of a bistable kinetic model with correlated noise [34]. Mei et al. considered the effects of cross-correlated white noise sources on the relaxation time and the correlation function of a bistable system [35,36]. They described the statistical properties of a bistable system with cross-correlation white noise sources. Considering two input signals that consist of an additive and a purely multiplicative random signal, Borromeo and Marchesoni investigated asymmetric probability densities in symmetrically modulated bistable devices [37,38]. In their work it is found that the correlation between an additive and a multiplicative noise plays an important role in the processes of a nonlinear stochastic system. Recently, as the subject matures, attention has turned to stochastic systems with cross-correlation additive and multiplicative coloured noise sources. Ling et al. investigated the
moments of the intensity of a single-mode laser driven by additive and multiplicative coloured noise sources with coloured cross-correlation [39]. Jin et al. [40] consider the relaxation time of a single-mode dye laser system driven by cross-correlation additive and multiplicative white noise sources.

The purpose of this work is to consider the effects of the cross-correlated coloured noise sources on the associated relaxation time, and on the intensity correlation function of a bistable system. In Section 2, the approximate Fokker-Planck equation (AFPE) is introduced for a bistable system with cross-correlation additive and multiplicative coloured noise sources. This is solved the AFPE for the stationary probability distribution (SPD). By using the projection operator method — in which the effects of the memory kernels are taken into account — the analytic expressions of the associated relaxation time and the normalized correlation function of a bistable system with cross-correlation coloured noise sources are derived. In Section 3, based on the numerical results, the relaxation time and the correlation function, the effects of the cross-correlation strength and the correlation time, and the stochastic resonant activation for the bistable system, are discussed. Thus, the important effects of cross-correlation coloured noise sources to the statistical properties of a bistable system are demonstrated.

2 Stationary probability distribution, relaxation time, and correlation function

Consider a conventional, symmetric, bistable, kinetic system driven by cross-correlation additive and multiplicative coloured noise, in which characteristics of the cross-correlation time and the self-correlation time of the noise sources may be different. The Langevin equation of this general system is

\[
\frac{dx}{dt} = x - x^3 + x\xi(t) + \eta(t).
\]

Here \(\xi(t)\) and \(\eta(t)\) are zero-mean Gaussian noise sources, whose statistical properties are

\[
\langle \xi(t) \rangle = \langle \eta(t) \rangle = 0,
\]

\[
\langle \xi(t)\xi(t') \rangle = \frac{D}{\tau_1} \exp \left( -\frac{|t-t'|}{\tau_1} \right),
\]

\[
\langle \eta(t)\eta(t') \rangle = \frac{Q}{\tau_2} \exp \left( -\frac{|t-t'|}{\tau_2} \right), \quad \text{and}
\]

\[
\langle \xi(t)\eta(t') \rangle = \langle \eta(t)\xi(t') \rangle = \frac{\lambda \sqrt{DQ}}{\tau_3} \exp \left( -\frac{|t-t'|}{\tau_3} \right),
\]

where \(D\) and \(Q\) are the multiplicative coloured noise and the additive coloured noise intensity, respectively. \(\tau_1\) and \(\tau_2\) are the self-correlation time of the multiplicative noise and the additive noise, respectively. \(\tau_3\) is the cross-correlation time of the multiplicative and additive coloured noise sources.

By virtue of the Novikov theorem [41], Fox’s approach [42], and the ansatz of Hanggi et al. [43], the approximate Fokker-Planck equation corresponding to equation (1) with equations (2–5) is obtained [16,27,28]:

\[
\frac{\partial P(x,t)}{\partial t} = L_{FP} P(x,t),
\]

\[
L_{FP} = -\frac{\partial}{\partial x} f(x) + \frac{\partial^2}{\partial x^2} G(x),
\]

where

\[
f(x) = x - x^3 + \frac{Dx}{1 + 2\tau_1} + \frac{\lambda \sqrt{DQ}}{1 + 2\tau_3},
\]

and

\[
G(x) = \frac{Dx^2}{1 + 2\tau_1} + \frac{2\lambda \sqrt{DQ} x}{1 + 2\tau_3} + \frac{Q}{1 + 2\tau_2}.
\]

Note that since \(\tau_1 \geq 0, \tau_2 \geq 0, \) and \(\tau_3 \geq 0\) satisfy the approximate Fokker-Planck equation (6) when \(1 + 2\tau_1 > 0, 1 + 2\tau_2 > 0, \) and \(1 + 2\tau_3 > 0, \) there is no restriction on \(\tau_1, \tau_2, \text{and} \tau_3 \) [25]. Now consider the case of the self-correlation time and the cross-correlation time satisfying \(\tau_1 = \tau_2 = \tau_3 = \tau, \) then

\[
f(x) = x - x^3 + \frac{Dx}{1 + 2\tau} + \frac{\lambda \sqrt{DQ}}{1 + 2\tau},
\]

and

\[
G(x) = \frac{Dx^2}{1 + 2\tau} + \frac{2\lambda \sqrt{DQ} x}{1 + 2\tau} + \frac{Q}{1 + 2\tau}.
\]

The stationary probability distribution of the system can be obtained from equation (6) with equations (10, 11):

\[
P_{st}(x) = N \left( \frac{Dx^2}{1 + 2\tau} + \frac{2\lambda \sqrt{DQ} x}{1 + 2\tau} + \frac{Q}{1 + 2\tau} \right)^{\beta_1} \times \exp \left[ -\frac{1 + 2\tau}{2D} x^2 + 2\lambda (1 + 2\tau) \sqrt{\frac{Q}{D^3}} x \right]
\]

\[
\times \exp \left[ \beta_2 \arctan \left( \sqrt{\frac{\sqrt{D} \lambda}{\sqrt{Q}/D}} \right) \right]
\]

for \(0 \leq |\lambda| < 1; \) and

\[
P_{st}(x) = N \left( \frac{Dx^2}{1 + 2\tau} + \frac{2\lambda \sqrt{DQ} x}{1 + 2\tau} + \frac{Q}{1 + 2\tau} \right)^{\beta_1} \times \exp \left[ -\frac{1 + 2\tau}{2D} x^2 + 2\lambda (1 + 2\tau) \sqrt{\frac{Q}{D^3}} x \right]
\]

\[
\times \exp \left[ \frac{-(1 + 2\tau)}{Dx + \lambda \sqrt{DQ}} \right]
\]

for \(|\lambda| = 1, \) where

\[
\beta_1 = \frac{1 + 2\tau}{2D} \left[ 1 + \frac{Q}{D} (1 - 4\lambda^2) \right] - \frac{1}{2},
\]

\[
\beta_2 = \frac{\lambda (1 + 2\tau)}{D \sqrt{1 - \lambda^2}} \left[ 1 + \frac{3Q}{D} \left( \frac{4\lambda^2}{D} \right) \right].
\]