Spin polarization effect in 2D and Q2D electron gas

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Abstract. We use the constructed spin-dependent static local field functions to calculate the plasmon dispersion of two-dimensional spin polarized electron gas (2D SPEG) over a range of electron densities at arbitrarily spin polarization. We also investigate how the finite width of electron layer will affect the plasmon frequency and inverse static dielectric function of 2D SPEG. Our results show that the effect of finite thickness on plasmon dispersion and inverse dielectric function becomes considerable even at high densities in 2D SPEG.

1 Introduction

A great deal of theoretical and experimental study has been carried out to investigate correlation effects in low-dimensional electron systems in particular two-dimensional electron gas (2DEG) [1–3]. This is due to availability of technological advances in fabrication techniques of 2D electron systems formed in semiconductor devices.

More over a simple model of diluted magnetic semiconductor heterostructures like CdMnTe and CdMgTe [4] is a spin-polarized electron gas (SPEG) in the presence of a static magnetic field. The field induces an equilibrium imbalance polarization \( \zeta = \frac{n_1 - n_0}{n_1 + n_0} \) in the number of electrons of opposite spins. The response to an external disturbance of coupled charge and spin fluctuations is characterized by appropriate susceptibility functions.

An important aspect of 2D electron systems is the plasma excitation, which is collective mode of charge density oscillation in the free carrier system. Many studies have been devoted to the dispersion and damping properties of 2DEG for several years [5–10] but to our knowledge they have not been fully analyzed in SPEG where the spin polarization effects become important. Therefore, the purpose of this paper is to determine the dispersions and nature of collective spin excitations in a 2D SPEG from the response functions.

On the other hand, at finite values of polarization, the exchange between parallel-spin electrons and correlations from the Coulomb repulsion are crucial in 2D systems. These effects are embedded in the many-body theory through the so-called local field corrections in the expression of the charge and spin susceptibilities of EG.

For this purpose we use approximate expressions for the static local field factor of a 2D SPEG making use of exact asymptotic behaviors obtained recently by present authors [11].

In order to have a more realistic description of the 2D electron systems we take into account the finite width of the EG. In this sense, electrons in a semiconductor inversion layer or in a quantum well can be modelled by a quasi-2DEG (Q2DEG) whose spin dependent conduction in solid-state devices has gained considerable importance in recent years.

Another practical function of 2DEG is the static (longitudinal) dielectric function that determines the response to a weak external perturbation. Since the electron correlation effects are clearly reflected in the dielectric function of 2D SPEG we also calculate the inverse static dielectric function for both 2D and Q2D SPEG at arbitrarily spin polarization for various electron densities.

2 Theoretical concept

The 2DEG is considered by usual dimensionless Wigner-Seitz density parameter \( r_s = (\pi n a_B^2)^{-1/2} \), \( a_B = \frac{\sqrt{3} K}{m^*} \) being the Bohr radius in the medium of interest with \( K \) and \( m_B \) appropriate dielectric constant and bare band mass respectively. The electrons interact via a coulomb potential \( v_q = \frac{2e^2}{Kq} \) and the charge response of the EG to an external perturbation is described in the linear regime by the susceptibilities \( \chi_{\sigma\sigma'}(q, \omega) \), the subscripts \( \sigma \) and \( \sigma' \) being spin indices (\( \uparrow \) or \( \downarrow \)) which can be obtained by the double Fourier transform of the correlation function

\[
K_{\sigma\sigma'}(|\mathbf{r} - \mathbf{r}'|, t - t') = -i\hbar \delta(t - t') \langle \hat{n}_{\sigma}(\mathbf{r}, t) \hat{n}_{\sigma'}(\mathbf{r}', t') \rangle.
\]
Here \( \theta(t) \) is the Heaviside step function, \( \langle \cdots \rangle \) denotes an average over the equilibrium ensemble, and \( \hat{n}_\sigma(r, t) \) is the spin density operator in the Heisenberg representation [2]. More explicitly, the expression for the linear susceptibilities is

\[
\chi_{\sigma\sigma'}(q, \omega) = \frac{\chi_0(\omega)}{\Delta(q, \omega)} \left[ \delta_{\sigma\sigma'} + \eta_{\sigma\sigma'} V^{\text{eff}}_\sigma(q) \chi_{0\sigma}(q, \omega) \right]
\]

(2)

where \( \eta_{\sigma\sigma'} = (-1)^{\delta_{\sigma\sigma'}} \), \( \chi_{0\sigma}(q, \omega) \) is the spin dependent Stern response function of a noninteracting 2DEG [1]:

\[
\chi_{0\sigma}(q, \omega) = \frac{1}{\Omega} \sum_k \frac{n_{k,\sigma} - n_{k+q,\sigma}}{\hbar \omega + E_{k,\sigma} - E_{k+q,\sigma}}.
\]

(3)

Here \( \Omega \) is the volume of the system, \( E_{k,\sigma} = \hbar^2 k^2 / 2m - \gamma \text{sign}(\sigma) B \) is the single particle energy in the static magnetic field \( B \) for an electron with statistical distribution \( n_{k,\sigma} \). The quantity \( \gamma = g \mu_B / 2 \) is the effective gyromagnetic factor where \( g \) is the Landé factor and \( \mu_B \) the Bohr magneton. The function \( \Delta(q, \omega) \) possesses information on the many-body dynamics of the EG and is given by

\[
\Delta(q, \omega) = \left[ 1 - V^{\text{eff}}_{\uparrow\uparrow}(q, \omega) \right] \left[ 1 - V^{\text{eff}}_{\downarrow\downarrow}(q, \omega) \right] - V^{\text{eff}}_{\uparrow\downarrow}(q) V^{\text{eff}}_{\downarrow\uparrow}(q, \omega) \chi_0(q, \omega) \chi_0(q, \omega).
\]

(4)

Here \( V^{\text{eff}}_{\sigma\sigma'}(q) = v_q [1 - G_{\sigma\sigma'}(q)] \) is the static effective spin-dependent coulomb interaction where \( G_{\sigma\sigma'}(q) \) describes Pauli and Coulomb holes around each electron within the system.

The plasmon dispersion \( \omega_{pl}(q) \) of the 2DEG can be obtained readily from the pole of \( \chi_{\sigma\sigma'}(q, \omega) \), i.e., by solving the equation \( \Delta(q, \omega) = 0 \). It is then clear that in order to calculate the plasmon energies of the system one has to obtain \( G_{\sigma\sigma'}(q) \).

As shown recently in our previous work [11], we model the same \( G_{\sigma\sigma} \) and opposite \( G_{\sigma\sigma'} \) spin effects by using spin-dependent local field functions \( G^+_\sigma(q, \omega) \) and \( G^r_\sigma(q, \omega) \) respectively:

\[
G^\pm_\sigma(q, \omega) = G_{\sigma\sigma}(q, \omega) \pm G_{\sigma\sigma'}(q, \omega).
\]

(5)

Following reference [11], we make use of approximate expressions for the spin-dependent local field correlation function in a 2D SPEG by their limiting behavior at small and large wave vectors and sum rules.

On the other hand, to study the effect of layer thickness on plasma excitation of the 2DEG at various densities, \( r_s \), and polarization degree, \( \xi \), we thus renormalize the bare Coulomb potential by means of a form factor to take into account the finite width of the EG in the GaAs/AlGaAs heterojunction-insulated gate field-effect transistor used in reference [12]. In reference [12] the finite width is introduced via a wave function of the form:

\[
\varphi(z) = (b^2/2)^{1/2} z \exp(-bz/2),
\]

(6)

where \( z \) is along the confinement direction and the adjustable parameter \( b \) is chosen to minimize the total single-particle energy

\[
E = \frac{\hbar^2 b^2}{8m} + \frac{12 \pi^2}{Kscb} \left( n_{\text{depl}} + \frac{11}{32} \right).
\]

(7)

In equation (7) the first and second terms are the contributions to the kinetic energy and electrostatic energy respectively. Thus, the appropriate renormalized potential in equation (2) for \( V^{\text{eff}}_{\sigma\sigma'}(q) \) is given by \( v_q \rightarrow v_q F(qd)/K \), where

\[
F(x) = 1 + \frac{K_{\text{ins}}}{K_{sc}} \frac{8 + 9x + 3x^2}{8(1+x)} + \frac{1}{2(1+x)^6}.
\]

(8)

with \( d = [\hbar^2 K_{sc}/(48\pi ne^2 n^*s)]^{1/2} \) representing an effective width of the 2DEG [8]. Here \( K_{\text{ins}} = 10.9 \) and \( K_{sc} = 12.9 \) are the dielectric constants of the insulator and of the space charge layer, \( m \) is the band mass in the confinement direction, and \( n^* = n_{\text{depl}} + (11/3) n \), the depletion layer charge density \( n_{\text{depl}} \) being zero in our calculation.

3 Results and conclusions

In Figure 1, \( G_{\sigma\sigma'}(q) \) are shown for a 2D electron gas with \( r_s = 5 \) and \( \xi = 0, 0.2, 0.4 \) and 0.8 as a function of normalized wave vector \( q/k_F \). It can be concluded from Figure 1 that the \( G_{\sigma\sigma'}(q) \) have a rather weak dependence on spin polarization \( \xi \) for fixed value of \( r_s \). Values of \( G_{\sigma\sigma'}(q) \) lying between zero and one reflect the depletion of charge density around each electron in the system. The intermediate value for electron density, \( r_s = 5 \), corresponds to a charge density of \( n = 1.22 \times 10^{-10} \text{ cm}^2 \) in GaAs.

Figure 2 shows the plasmon dispersion curves of a 2DEG as a function of \( q/k_F \) for \( r_s = 1, 2, 3 \) and 5 for polarization degrees \( \xi = 0 \) and 0.8. It should be noted that, for the 2D SPEG, the boundary condition for the single electron-hole pair excitations at zero temperature is given by

\[
\hbar \omega_{\text{eh}}(q) = \hbar v_{F,\sigma} q + \hbar k^2 q^2 / 2m,
\]

(9)

where we have introduced the spin-resolved Fermi velocities and wave vectors, \( v_{F,\sigma} = \hbar K_{F,\sigma} / 2m \), \( K_{F,\sigma} = k_F \sqrt{1 + \text{sign}(\sigma)} \). Here \( \text{sign}(\sigma) \) is equal to 1 (or 0) when...