CP violation in the general two-Higgs-doublet model: a geometric view

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Abstract We discuss the CP properties of the potential in the general two-Higgs-doublet model (THDM). This is done in a concise way using real gauge invariant functions built from the scalar products of the doublet fields. The space of these invariant functions, parametrising the gauge orbits of the Higgs fields, is isomorphic to the forward light cone and its interior. CP transformations are shown to correspond to reflections in the space of the gauge invariant functions. We consider CP transformations where no mixing of the Higgs doublets is taken into account as well as the general case where the Higgs basis is not fixed. We present basis independent conditions for explicit CP violation which may be checked easily for any THDM potential. Conditions for spontaneous CP violation, that is CP violation through the vacuum expectation values of the Higgs fields, are also derived in a basis independent way.

1 Introduction

In the Standard Model (SM) and in many extensions of it like the Minimal Supersymmetric Standard Model (MSSM) [1, 2] the electroweak symmetry breaking is accomplished via the Higgs mechanism. In the SM, where one Higgs doublet is introduced, the Higgs potential is automatically invariant under CP transformations. Thus, CP violation in the SM only arises via Yukawa interactions of the Higgs field with the fermions, that is, through the Kobayashi–Maskawa mechanism [3].

Here we investigate models having the standard weak isospin times hypercharge $(SU(2)_L \times U(1)_Y)$ gauge group as invariance group and a Higgs sector with two doublets. That is, we consider the general Two-Higgs-Doublet Model (THDM). In contrast to the SM, in the THDM the Higgs potential itself is in general not invariant under CP transformations [3].

The CP properties of the Higgs potential are studied in the framework of gauge invariant functions, built from all possible $SU(2)_L \times U(1)_Y$ invariant scalar products of Higgs doublets [4]. In this approach all invariant scalar products are replaced by real gauge invariant functions which can be combined to a four-vector. In terms of these real gauge invariant functions, a mixing of the Higgs doublets corresponds to rotations of the space-like components of this four-vector and, as we shall show, CP transformations correspond to reflections of the space-like components. Thus, constraints for CP invariance can be derived concisely in this geometric picture. We also give unambiguous criteria for the occurrence of spontaneous CP violation, where CP violation arises from the vacuum expectation values of the Higgs doublets, although the Higgs potential itself is CP invariant.

There is much interest in the investigation of an extension of the Higgs sector for several reasons: supersymmetric extensions require one to have at least two Higgs doublets in order to give masses to up- and down-type fermions and to keep the theory anomaly free. Generally, the naturality problem arising in the SM is crucially depending on the Higgs sector. In [5] this has been used as a motivation to focus on the THDM. For a recent proposal of THDMs having a custodial symmetry, see [6]. Another reason, originating from cosmology, is that CP violation is one of the three Sakharov criteria which have to be fulfilled in order to explain the observed baryon–antibaryon asymmetry in our Universe through the particle dynamics [7, 8]. In the SM, given the strength of the observed CP violation and the experimental lower bound on the Higgs mass, one cannot explain the baryon excess over anti-baryons observed in our Universe. For a review see for instance [9]. A possible way out of this dilemma is to consider models with an extended Higgs sector.

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There exists already an extensive literature on CP violation in multi-Higgs and, in particular, two-Higgs-doublet models. A general discussion of CP transformations in gauge theories was given in [10]. In [11, 12] basis independent conditions for spontaneous CP violation are given for the general THDM. References [13, 14] provide an extensive analysis of the general THDM in terms of invariants with respect to \( U(2) \) Higgs basis changes. In [13] a proof is given that the conditions of [11] for spontaneous CP violation are sufficient and necessary. Reference [14] determines the necessary and sufficient conditions for explicit CP violation in a basis independent way via the systematic check of potentially complex invariants. A rather detailed account of CP violation in N-Higgs-doublet models in general and THDMs in particular was given in [15] using gauge invariant functions. In [17] the Higgs mass squared matrix is considered and CP-conservation conditions are determined from the possible mixing of CP-even and CP-odd entries in this matrix. Reference [16] is devoted to spontaneous symmetry breaking in THDMs, focusing critically on the issue if and when the usual parameter \( \tan \beta \) can be considered to be a truly physical parameter. A measure for CP violating effects is discussed in [18] for a given Higgs basis and vacuum. Let us also mention the investigation of the minima structure of THDMs in context with CP violation; see [19] and references therein. In [20] the THDM was studied from a group theoretic point of view. In [21, 22] the Minkowski space structure of the \( \mathbf{K} \)-space (in our notation) was emphasised. Lorentz transformations were used to diagonalise the term of the potential \( V \) (24) quadratic in \( \mathbf{K} \). In our present paper we have not used Lorentz transformations in \( \mathbf{K} \)-space for several reasons. Lorentz transformations do in general not respect the form of the kinetic term in the Higgs–Lagrangian. In [23] we are interested in the complete theory. Thus we only consider Higgs-basis transformations which keep the kinetic term invariant. There are potentials which are stable in the weak sense (see Sect. 4 of [4]) and thus completely acceptable from a physical point of view. We find examples of such potentials where the term quadratic in \( \mathbf{K} \) cannot be diagonalised by a Lorentz transformation. In our work we do not exclude these cases from the discussion. Also we find it generally advantageous to give criteria for properties of a THDM in a way directly applicable for any given model without assuming a particular choice for the Higgs-flavour basis.

In our present paper we take up again the question of CP violation in THDMs. We derive some new results and rederive already known results in a way as we need it for the companion paper [23]. Indeed, the present paper and [23] should be considered as belonging together and forming one unit. Our present paper is organised as follows. In Sect. 2 we briefly recall the definitions of the gauge invariant functions which provide our framework to investigate CP properties. Then, in Sect. 3, we classify the possible types of CP transformations and present constraints for CP invariance of the potential in this framework. This is followed in Sect. 4 by a discussion of spontaneous CP violation. The general results are illustrated in Sect. 5, where we discuss two specific models in the more conventional parametrisation of [24]. Section 6 contains our conclusions. In the respective sections we also compare our findings to those in the literature mentioned above. The appendices contain the proofs of two theorems and details for general models with different types of CP symmetries.

2 Gauge invariant functions in the general two-Higgs-doublet model

We shall use the gauge invariant functions as introduced in [4]. Here we recall the formalism briefly in order to make this work self-contained.

We denote the two complex Higgs-doublet fields by

\[
\phi_i(x) = \begin{pmatrix} \phi_i^1(x) \\ \phi_i^2(x) \end{pmatrix}
\]

with \( i = 1, 2 \). Hence we have eight real scalar degrees of freedom. The most general \( SU(2)_L \times U(1)_Y \) invariant Lagrangian for the THDM can be written as

\[
\mathcal{L}_{\text{THDM}} = \mathcal{L}_\psi + \mathcal{L}_{\text{Yuk}} + \mathcal{L}_V,
\]

where the Higgs-boson Lagrangian is given by

\[
\mathcal{L}_\psi = \sum_{i=1,2} (\bar{D}_\mu \phi_i)(D^\mu \phi_i) - V(\phi_1, \phi_2).
\]

This term replaces the kinetic terms of the Higgs boson and the Higgs potential in the SM Lagrangian. The covariant derivative is

\[
D_\mu = \partial_\mu + ig W^a_\mu T_a + ig' B_\mu Y,
\]

where \( T_a \) and \( Y \) are the generating operators of weak-isospin and weak-hypercharge transformations. For the Higgs doublets we have \( T_a = \tau_a / 2 \), where \( \tau_a \) (\( a = 1, 2, 3 \)) are the Pauli matrices. We assume both doublets to have weak hypercharge \( \gamma = +1/2 \). By \( \mathcal{L}_{\text{Yuk}} \) we denote the Yukawa-interaction terms of the Higgs fields with the fermions. Finally, \( \mathcal{L}_V \) contains the terms of the Lagrangian without Higgs fields. We do not specify \( \mathcal{L}_{\text{Yuk}} \) and \( \mathcal{L}_V \) here since they are not relevant for our analysis.

We remark that in the MSSM the two Higgs doublets \( H_1 \) and \( H_2 \) carry hypercharges \( \gamma = -1/2 \) and \( \gamma = +1/2 \), respectively, whereas here we use the conventional definition