Unraveling AdS/CFT

What string theory is telling us about the physics at RHIC

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Abstract In this review, I survey the conjectured correspondence between a string theory in ten dimensions and certain supersymmetric gauge theories in four. This duality has recently garnered considerable attention from scientists studying the hot matter produced in heavy-ion collisions. An important and immediate question is to what extent one can hope to describe the dynamics of the quark–gluon plasma in a supersymmetric conformal field theory. Here I explain recent applications of the AdS/CFT correspondence to the strongly interacting matter produced at the Relativistic Heavy Ion Collider. Progress in characterizing the medium with these techniques will be discussed, as well as limitations inherent to the method.

1 Introducing the correspondence

The prevailing theory of interacting quarks and gluons, quantum chromodynamics, is a phenomenologically rich, yet theoretically taxing description of fundamental particles. One of the principal difficulties is the fact that the renormalization group equations describing QCD indicate confinement in the infrared and asymptotic freedom in the UV. While well-known perturbative techniques exist for studying matter in the regime where the strong coupling constant, \(\alpha_s\), is small, there is presently no exact solution to the theory at sufficiently large \(\alpha_s\).

The study of partons interacting at strong coupling (\(\alpha_s \gtrsim 1/2\)) has gained considerable attention due to recent experimental results from the Relativistic Heavy Ion Collider (RHIC). Specifically, the success of hydrodynamic models in describing the elliptic flow in the medium as well as the observed magnitude of the ratio of the viscosity to entropy density (\(\eta/s \sim 0.03–0.1\) [1]) favor a picture of the quark–gluon plasma as a strongly interacting fluid over one of weakly interacting quark and gluon degrees of freedom.

Until about a decade ago, theoretical descriptions of gauge theories at strong coupling were usually formulated on the lattice (in Euclidean time), which limited their applicability to thermodynamic quantities like densities and susceptibilities. Another option (the 't Hooft approach) is to try to solve the SU\((N_c)\) theory exactly in the limit \(N_c \to \infty\) with fixed 't Hooft coupling, \(\lambda = g_{YM}^2 N_c\), and hope for an expansion in \(1/N_c\). In the late nineties, a somewhat surprising technique was discovered for studying gauge theories at large \(\lambda\). This method, generically labeled the anti-de Sitter/conformal field theory correspondence (AdS/CFT) relates a field theory in \(3 + 1\) dimensions to a string theory in \(9 + 1\) dimensions [2]. This duality and its application to heavy-ion collisions are the focus of these proceedings.

1.1 The way things work (part I)

To motivate the correspondence, it is useful to consider the particular arrangement of strings and D-branes shown in Fig. 1.1. Strings are objects extended along one spatial dimension, and they can be thought of in the present case as

Fig. 1.1 A stack of \(N_c\) coincident D-branes (the colored planes) realizes an SU\((N_c)\) supersymmetric gauge theory on its world volume. The string excitations correspond to adjoint degrees of freedom (gluons)

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excitations of branes. Dp-branes live in p spatial dimensions and can be thought of as boundaries on which strings can end. Here I consider only the case where p = 3, so that a stack of coincident branes has a 4-dimensional worldvolume.

The approach taken here will be to study this configuration of \( N_c \) branes and its excitations. Since each string endpoint is confined to a brane, it is clear that there are \( N_c^2 \) ways in which a string may be situated on the stack. As has been known for some time [3], these \( N_c^2 \) degrees of freedom correspond precisely to the gauge bosons of an \( SU(N_c) \) super Yang–Mills (SYM) theory. \( \mathcal{N} = 4 \) SYM. In this way, one obtains a description of the stack of branes in terms of the 3 + 1 dimensional gauge theory that lives on its worldvolume.

As suggested previously, one might hope to learn something about the stack of branes (and its \( SU(N_c) \) gauge theory) by taking the 't Hooft limit in which the number of colors becomes very large. In the brane picture, this corresponds to throwing a large number of D3-branes onto the stack. These branes are massive, and when many of them are piled atop one another, it is no longer tenable to ignore their back reaction on the spacetime around them. This suggests a second, distinct description of the stack of branes in terms of a string theory in a curved spacetime.

With these two descriptions of coincident D-branes at hand, one arrives at the fundamental insight of AdS/CFT: if the physics of the branes can be described in terms of both the gauge theory (CFT) on the stack’s worldvolume and by a string theory in a curved background (AdS), then there should be some dictionary that allows one to translate parameters in one description into parameters in the other.

In practice, the AdS/CFT correspondence is most useful for studying the properties of gauge theories at strong coupling. To see this, consider the following entry from the AdS/CFT dictionary:

\[
\frac{R^4}{l_s^4} = \lambda. \tag{1}
\]

Here the parameters on the left hand side are those of the string theory. Specifically, \( R \) is the gravitational radius of \( \text{AdS}_5 \times \text{S}_5 \), while \( l_s \) is the string length. On the right hand side, \( \lambda \) is the 't Hooft coupling of the gauge theory. Evidently, at strong coupling (\( \lambda \gg 1 \)) the dual geometry is described by a gravitational radius much larger than the string length. In this regime, string theory is well approximated by classical supergravity, and it is in general easier to study.

1.2 The way things work (part II)

While the sketch offered in Sect. 1.1 was rather rough, it nevertheless suggests an immediate question one might hope to answer: if some strongly coupled gauge theory is described by classical strings in some background, what spacetime looks like hot QCD?

While this question is in principle very difficult to answer, one may nevertheless enumerate some desirable properties the background should possess. To be a viable dual for a quark–gluon plasma, one might look for geometries that have parameters like temperature and entropy, and maybe some sort of hydrodynamic behavior. In fact, the string equations of motion do permit such a solution. These geometries have been known for many years, and are called “black \( p \)-branes”. Not surprisingly, they are analogous to the well-known Schwarzschild black hole, the principal difference being a horizon that extends along \( p \) spatial dimensions:

\[
ds^2 = \frac{r^2}{T^2} \left[ -f(r) \, dt^2 + d\vec{x}^2 \right] \\
+ \frac{R^2}{r^2} f(r)^{-1} \, dr^2 + R^2 \, d\Omega_5^2,
\tag{2}
\]

where \( f(r) = 1 - \frac{r_H^4}{r^4} \), \( r_H \) is the location of the horizon along the radial direction, and \( d\Omega_5^2 \) is the metric of the unit 5-sphere. One may study this metric with the standard techniques, computing (for example) the Hawking temperature by Wick rotating and ensuring that no conical singularity develops. Performing this simple analysis, one finds \( T_H = \frac{r_H}{\pi} R^2 \).

Identifying this temperature with the temperature of the field theory, one finds that raising the horizon corresponds to heating up the SYM plasma. This is illustrated schematically in Fig. 1.2.

![Fig. 1.2 Sketch of the geometry dual to \( \mathcal{N} = 4 \) at finite temperature. The flat 9 + 1 dimensional Minkowski space atop the throat is discarded in the correspondence; all the action takes place in the asymptotically AdS throat. The location of the horizon acts as a thermostat in the dual gauge theory.](Image 357x135 to 492x261)

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1In fact, \( N_c^2 \) is the dimension of the adjoint for a \( U(N_c) \) theory. Through the familiar process \( U(N_c) \cong SU(N_c) \otimes U(1) \) the \( U(1) \) decouples, and the number of gluons is reduced to \( N_c^2 - 1 \).