Asymmetry parameter of $K_1(1270, 1400)$ by analyzing the $B \to K_1 \bar{\nu} \nu$ transition form factors within QCD

M. Bayar\textsuperscript{a}, K. Azizi\textsuperscript{b}

Physics Department, Middle East Technical University, 06531 Ankara, Turkey

Received: 14 November 2008 / Revised: 10 January 2009 / Published online: 8 April 2009 © Springer-Verlag / Società Italiana di Fisica 2009

Abstract Separating the mixture of the $K_1(1270)$ and $K_1(1400)$ states, the $B \to K_1(1270, 1400)\nu\bar{\nu}$ transition form factors are calculated in the three-point QCD sum rule approach. The longitudinal, transverse and total decay widths as well as the asymmetry parameter, characterizing the polarization of the axial $K_1(1270, 1400)$ and the branching ratio for these decays, are evaluated.

PACS 11.55.Hx · 13.20.He

1 Introduction

The $B \to K_1(1270, 1400)\nu\bar{\nu}$ transitions are governed by the flavor changing neutral current (FCNC) decay of $b \to s\nu\bar{\nu}$, which is of fundamental interest because of the following reasons: such a transition occurs at loop level; and it is forbidden at tree level in the Standard Model (SM). This transition is a good candidate for searching new physics beyond the SM and constrains the parameters beyond it. A search for SUSY particles [1], light dark matter [2] and also a fourth generation of the quarks is possible by analyzing such loop level transition. The $B \to K_1(1270, 1400)\nu\bar{\nu}$ decays also provide a new framework for precise calculation of the $V_{tb}$ and $V_{ts}$ as elements of the Cabibbo–Kobayashi–Maskawa (CKM) matrix.

Experimentally, the $K_1(1270)$ and $K_1(1400)$ are mixtures of the strange members of two axial-vector $SU(3)$ octets $^3P_1(K_1^A)$ and $^1P_1(K_1^B)$. The $K_1(1270, 1400)$ and $K_1^{A,B}$ states are related to each other by [3–5]

\begin{align}
|K_1(1270)| &= |K_1^A| \sin \theta + |K_1^B| \cos \theta, \\
|K_1(1400)| &= |K_1^A| \cos \theta - |K_1^B| \sin \theta,
\end{align}

the angle $\theta$ lies in the interval $37^\circ \leq \theta \leq 58^\circ$, $-58^\circ \leq \theta \leq -37^\circ$ [3–8]. The sign ambiguity for the mixing angle is related to the fact that one can add an arbitrary phase to the $|K_1^A|$ and $|K_1^B|$. In recent studies for $B \to K_1(1270)\nu$ and $\tau \to K_1(1270)\nu$, the following values have been obtained for $\theta$ [9], which we are going to use in the present work:

\begin{equation}
\theta = -(34 \pm 13)^\circ.
\end{equation}

$B \to K_1\gamma$ decay has been investigated in the next-to-leading order in the large energy effective theory (LEET) and in the framework of light cone sum rules in [10] and [3, 4], respectively. In [11], the $B \to K_1(1270)\ell\bar{\nu}$ transition has also been investigated in the LEET model. In this work, separating the $K_1(1270)$ and $K_1(1400)$ states, we analyze the $B \to K_1(1270, 1400)\nu\bar{\nu}$ decay modes in the framework of the three-point QCD sum rules. First, we calculate the form factors of $B$ to the axial $|K_1^A|$ and $|K_1^B|$ states. Then, using the relations among the form factors of the $K_1(1270), K_1(1400), |K_1^A|$ and $|K_1^B|$, we calculate the form factors of the $B \to K_1(1270)$ and $B \to K_1(1400)$ transitions.

The transition form factors play a fundamental role in evaluating of the longitudinal, transverse and total decay widths as well as the asymmetry parameter of $K_1(1270, 1400)$. For calculating these form factors, we use the well established QCD sum rules method as a non-perturbative method based on the fundamental QCD Lagrangian.

The paper encompasses three sections. In Sect. 2, the form factors of the $B \to K_1^{A(B)}\nu\bar{\nu}$ transition as well as the longitudinal and transverse component of the decay width and asymmetry parameter for $K_1(1270, 1400)$ are calculated. Section 3 is devoted to the numerical analysis and discussions.

\textsuperscript{a}e-mail: mbayar@metu.edu.tr
\textsuperscript{b}e-mail: e146342@metu.edu.tr
2 Sum rules for the $B \rightarrow K_1^{A(B)} \nu \overline{\nu}$ transition form factors

At the quark level, the $B \rightarrow K_1^{A(B)} \nu \overline{\nu}$ decay proceeds by the loop $b \rightarrow s \nu \overline{\nu}$ transition. The Hamiltonian responsible for this transition is given by

$$ H_{\text{eff}} = \frac{G_F \alpha_{\text{em}}}{2\sqrt{2\pi}} C_{10} V_{tb} V_{ts}^* \tau \gamma_\mu (1 - \gamma_5) v \overline{s} \gamma_\mu (1 - \gamma_5) b. \quad (3) $$

To obtain the transition amplitude for $B \rightarrow K_1^{A(B)} \nu \overline{\nu}$ decay, it is necessary to sandwich (3) between the initial and final meson states:

$$ M = \frac{G_F \alpha_{\text{em}}}{2\sqrt{2\pi}} C_{10} V_{tb} V_{ts}^* \tau \gamma_\mu (1 - \gamma_5) v \langle K_1^{A(B)} (p', \varepsilon) | \overline{s} \gamma_\mu (1 - \gamma_5) b | B(p) \rangle, \quad (4) $$

where $G_F$ is the Fermi constant, $\alpha_{\text{em}}$ is the fine structure constant at the Z mass scale and $V_{ij}$ are the elements of the CKM matrix. Both vector and axial-vector part of the transition current, $\tau \gamma_\mu (1 - \gamma_5) b$, contribute to the matrix element stated in (4). Considering the Lorentz and parity invariances, this matrix element can be parameterized in terms of some form factors as follows:

$$ \langle K_1^{A(B)} (p', \varepsilon) | \overline{s} \gamma_\mu b | B(p) \rangle = i \left[ f_0^{A(B)} (q^2) (m_B + m_{K_1^{A(B)}}) \varepsilon_\mu^* \right. \left. - f_+^{A(B)} (q^2) (m_B + m_{K_1^{A(B)}}) (\varepsilon^* p) P_\mu \right. \left. - f_-^{A(B)} (q^2) (m_B + m_{K_1^{A(B)}}) (\varepsilon^* p) q_\mu \right]. \quad (5) $$

$$ \langle K_1^{A(B)} (p', \varepsilon) | \overline{s} \gamma_\mu \gamma_\nu b | B(p) \rangle = - f_V^{A(B)} (q^2) \varepsilon_{\mu\nu\alpha\beta} \varepsilon^{* \nu} p^\alpha p'^\beta, \quad (6) $$

where $f_V^{A(B)} (q^2)$, $f_0^{A(B)} (q^2)$, $f_+^{A(B)} (q^2)$ and $f_-^{A(B)} (q^2)$ are the transition form factors and $P_\mu = (p + p')_\mu$, $q_\mu = (p - p')_\mu$. Here, we should mention that the $f_-^{A(B)} (q^2)$ form factor does not appear in the expressions of the decay widths, so we do not consider it in our calculations. Using (1), (5), (6) we obtain

$$ f_0^{B \rightarrow K_1(1270)} = \frac{m_B + m_{K_1^A}}{m_B + m_{K_1^B}} f_0^{B \rightarrow K_1^A} \sin \theta $$

$$ + \frac{m_B + m_{K_1^A}}{m_B + m_{K_1^B}} f_0^{B \rightarrow K_1^B} \cos \theta, \quad (7) $$

$$ f_0^{B \rightarrow K_1(1400)} = \frac{m_B + m_{K_1^A}}{m_B + m_{K_1^B}} f_0^{B \rightarrow K_1^A} \cos \theta $$

$$ - \frac{m_B + m_{K_1^A}}{m_B + m_{K_1^B}} f_0^{B \rightarrow K_1^B} \sin \theta, \quad (7) $$

$$ f_0^{B \rightarrow K_1(1270)} = \frac{m_B + m_{K_1^A}}{m_B + m_{K_1^B}} f_0^{B \rightarrow K_1^A} \sin \theta $$

$$ + \frac{m_B + m_{K_1^A}}{m_B + m_{K_1^B}} f_0^{B \rightarrow K_1^B} \cos \theta, \quad (8) $$

$$ f_0^{B \rightarrow K_1(1400)} = \frac{m_B + m_{K_1^A}}{m_B + m_{K_1^B}} f_0^{B \rightarrow K_1^A} \cos \theta $$

$$ - \frac{m_B + m_{K_1^A}}{m_B + m_{K_1^B}} f_0^{B \rightarrow K_1^B} \sin \theta. \quad (8) $$

For simplicity, we will set $f_i^{B \rightarrow K_1^{A(B)}} = f_i^{A(B)}$ in the future calculations.

We define the G-parity conserving decay constants of the axial-vector mesons $K_1^A$ and $K_1^B$ as

$$ \langle K_1^A (p', \varepsilon) | J_{\mu} = \bar{s} \gamma_\mu \gamma_5 u | 0 \rangle = - i f_{K_1^A} m_{K_1^A} \varepsilon_\mu, \langle K_1^B (p', \varepsilon) | J_{\mu} = \bar{s} \sigma_{\mu\nu} \gamma_5 u | 0 \rangle $$

$$ = f_{K_1^B}^\perp (1 \text{ GeV}) (\varepsilon_\mu p'_{\nu'} - \varepsilon_{\nu'} p'_\mu), \quad (9) $$

where $f_{K_1^A}$ is the scale independent decay constant of the $K_1^A$ meson; however, $f_{K_1^B}^\perp$ is the scale dependent leptonic constant of the $K_1^B$ meson. The $f_{K_1^B}^\perp$ is calculated at the scale 1 GeV. The decay constants $f_{K_1^A}$ and $f_{K_1^B}$ are calculated in the framework of the light cone QCD sum rules with the help of the distribution amplitudes of the axial $K_1^A$ and $K_1^B$ states in [12, 13]. On the other hand, the G-parity violating decay constants are defined as

$$ \langle K_1^A (p', \varepsilon) | J_{\mu \nu'} = \bar{s} \sigma_{\mu \nu'} \gamma_5 u | 0 \rangle $$

$$ = f_{K_1^A}^\perp m_{K_1^A}^{\perp} \varepsilon_\mu p'_{\nu'} - \varepsilon_{\nu'} p'_\mu, \quad (10) $$

$$ \langle K_1^B (p', \varepsilon) | J_{\mu} = \bar{s} \gamma_\mu \gamma_5 u | 0 \rangle $$

$$ = i f_{K_1^B}^\perp (1 \text{ GeV}) m_{K_1^B} \varepsilon_\mu, \quad (10) $$

$$ \langle K_1^B (p', \varepsilon) | J_{\mu} = \bar{s} \gamma_\mu \gamma_5 u | 0 \rangle $$

$$ = i f_{K_1^B}^\perp (1 \text{ GeV}) m_{K_1^B} \varepsilon_\mu, \quad (10) $$