Testing the equivalent photon approximation of the proton in the process $ep \rightarrow \nu WX$

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Received: 10 August 2004 / Published online: 3 November 2004 – © Springer-Verlag / Società Italiana di Fisica 2004

Abstract. The accuracy of the equivalent photon approximation of the proton in describing the inelastic process $ep \rightarrow \nu WX$ is investigated. In particular, the scale dependence of the corresponding inelastic photon distribution is discussed. Furthermore, an estimate of the total number of events, including the ones coming from the elastic and quasi-elastic channels of the reaction, is given for the HERA collider.

1 Introduction

The equivalent photon approximation (EPA) of the nucleon $N (= p, n)$ is a technical device which allows for a rather simple and efficient calculation of any photon-induced subprocess, whose cross section can be written as a convolution of the probability that the nucleon radiates off a photon (equivalent photon distribution) with the corresponding real photoproduction cross section. The polarized and unpolarized photon distributions of the nucleon, evaluated in the EPA, have been computed theoretically [1] and the possibility of their experimental determination has also been demonstrated [2–5]. Both of them consist of two components, an elastic one, due to $N \rightarrow \gamma N$, and an inelastic one, due to $N \rightarrow \gamma X$, with $X \neq N$. The reliability of the EPA remains, however, to be studied.

In [6] the unpolarized elastic photon distribution was tested in the case of $\nu W$ production in the process $ep \rightarrow \nu Wp$. The relative error of the cross section as calculated in the EPA with respect to the exact result was shown as a function of $\sqrt{S}$, in the range $100 \leq \sqrt{S} \leq 1800 \text{GeV}$. The agreement turned out to be very good, the approximation reproducing the exact cross section within less than one percent. Motivated by this results, our aim here is to check if the same holds in the inelastic channel.

The process $ep \rightarrow \nu WX$ has been widely studied by several authors [7–11]. Its relevance is related to the possibility of measuring the three-vector-boson coupling $WW\gamma$, which is a manifestation of the non-abelian gauge symmetry upon which the standard model is based. The observation of the vector-boson self-interaction would be a crucial test of the theory. Furthermore, such a reaction is also an important background to a number of processes indicating the presence of new physics. The lightest supersymmetric standard model particle has no charge and interacts very weakly with matter; this means that, exactly as the neutrino from the standard model, it escapes the detector unobserved and can be recognized only by missing momentum. This implies that a detailed study of the processes with neutrinos in the final states is necessary to distinguish between the new physics of the supersymmetric standard model and the physics of the standard model. At the HERA collider energies ($\sqrt{S} = 318 \text{GeV}$) the $ep \rightarrow \nu WX$ cross section is much smaller than the one for $ep \rightarrow eWX$ [8,9], also being sensitive to the $WW\gamma$ coupling due to the presence in the latter of an additional Feynman graph where an almost real photon and a massless quark are exchanged in a $u$-channel configuration ($u$-channel pole). The dominance of the process $ep \rightarrow eWX$ justifies the higher theoretical and experimental [12] attention that it has received so far, as compared to $ep \rightarrow \nu WX$. One way of improving the problem of the low number of deep inelastic $\nu W$ events at HERA would be to consider also the elastic and quasi-elastic channels of the reaction, as will be discussed in Sect. 3.

It is worth mentioning that not all the calculations of the $ep \rightarrow \nu WX$ event rates available in the literature, in which only the photon exchange is considered (see Fig. 1), are in agreement, as already pointed out in [11]. In particular, the numerical estimate of the cross section for HERA energies presented in [7,8], obtained in the EPA approach, is one half of the one published in [11], obtained within the framework of the helicity amplitude formalism without any approximation. The value given in [10] is even bigger than the one in [11]: all these discrepancies cannot be due to the slightly different kinematical cuts employed in the papers cited above and stimulate a further analysis. Our results agree with [7,8].

The plan of this paper is as follows. In Sect. 2 we calculate the exact cross section for the inelastic channel in a manifestly covariant way, and we show in which kinematical region it is supposed to be well described by the EPA. The formulae for the corresponding elastic cross sections,

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both the exact and the one evaluated in the EPA, are also given. The numerical results are discussed in Sect. 3. The summary is given in Sect. 4.

2 Theoretical framework

The $\nu W$ production from inelastic $ep$ scattering,

$$e(l) + p(P) \rightarrow \nu(l') + W(k') + X(P_X),$$

(1)

is described, considering only one photon exchange, by the Feynman diagrams depicted in Fig. 1. The four-momenta of the particles are given in the brackets; $P_X = \sum X P_X$ is the sum over all momenta of the produced hadronic system. We introduce the invariants

$$S = (P + l)^2, \quad \hat s = (l + k)^2, \quad \hat t = (l - l')^2, \quad Q^2 = -k^2,$$

(2)

where $k = P - P_X$ is the four-momentum of the virtual photon. Following [2], the integrated cross section can be written as

$$\sigma_{\text{inel}}(S) = \frac{\alpha}{4\pi(S - m^2)^2} \times \int_{W_{\text{min}}}^{W_{\text{max}}} \frac{dW}{\sqrt{\hat s}} \int_{\hat s_{\text{min}}}^{\hat s_{\text{max}}} \frac{d\hat s}{2\hat s} \int_{Q_{\text{min}}}^{Q_{\text{max}}} \frac{dQ^2}{Q^4} \int_{i_{\text{min}}}^{i_{\text{max}}} \frac{di}{2\pi} \int_{0}^{2\pi} d\varphi^* \times \left[ X_1(\hat s, Q^2, \hat t) + X_2(\hat s, Q^2, \hat t) \right]$$

$$\times F_2(x_B, Q^2) \frac{x_B}{2} - X_2(\hat s, Q^2, \hat t) F_1(x_B, Q^2),$$

(3)

where $W^2$ indicates the invariant mass squared of the produced hadronic system $X$, $\varphi^*$ denotes the azimuthal angle of the outgoing $\nu-W$ system in the $\nu-W$ CM frame, and

$$x_B = \frac{Q^2}{W^2 + Q^2 - m^2}$$

(4)

is the Bjorken variable. $F_{1,2}(x_B, Q^2)$ are the structure functions of the proton and the two invariants $X_{1,2}(\hat s, Q^2, \hat t)$, which contain all the information about the subprocess $e\gamma^* \rightarrow \nu W$, are given by

$$X_1(\hat s, Q^2, \hat t) = \frac{\alpha G_F}{2\sqrt{2\pi}} \frac{Q^2 M_W^4}{(Q^2 + \hat s)^3(M_W^2 - \hat t)^2} \times [(Q^2 + \hat s)^3 - \hat s(Q^2 + \hat s)^2(Q^2 + \hat s + \hat t) + 2(Q^2 + \hat s)^2 \hat t + 8(Q^2 + \hat s)\hat t^2 + 8\hat t^3]$$

(5)

and

$$X_2(\hat s, Q^2, \hat t) = \frac{\alpha G_F}{2\sqrt{2\pi}} \frac{1}{\hat s^2(\hat s + \hat t)(M_W^2 - \hat t)^2} \times \{ 4M_W^4 [(Q^2 + \hat s) - 4M_W^2 \hat t] + 4M_W^4 [4\hat t^2 + 8\hat t \hat s + 4M_W^2] \}$$

(6)

In (3) the minimum value of $\hat s$ is given by the squared mass of the $W$ boson:

$$\hat s_{\text{min}} = M_W^2,$$

(7)

while the limits of the integration over $W^2$ are

$$W_{\text{min}}^2 = (m + m_\pi)^2, \quad W_{\text{max}}^2 = (\sqrt{S} - \sqrt{\hat s_{\text{min}}})^2,$$

(8)

where $m_\pi$ is the mass of the pion. The limits $Q_{\text{min, max}}^2$ are given by

$$Q_{\text{min, max}}^2 = -m^2 - W^2 + \frac{1}{2\hat s} \left[ (S + m^2)(S - \hat s + W^2) \mp (S - m^2) \sqrt{(S - \hat s + W^2)^2 - 4SW^2} \right],$$

(9)

and the extrema of $\hat t$ are

$$\hat t_{\text{max}} = 0, \quad \hat t_{\text{min}} = -\frac{(\hat s + Q^2)(\hat s - M_W^2)}{\hat s}.$$

(10)

Integrating $X_{1,2}(\hat s, Q^2, \hat t)$ over $\varphi^*$ and $\hat t$, with the limits in (10), one recovers (4.1) and (4.2) of [6] respectively, times a factor of two due to a different normalization.