The phase of the $\sigma \rightarrow \pi\pi$ amplitude in $J/\Psi \rightarrow \omega\pi^+\pi^-$

D.V. Bugg

Queen Mary, University of London, London E1 4NS, UK

Received: 14 September 2004 / Published online: 20 October 2004 – © Springer-Verlag / Società Italiana di Fisica 2004

Abstract. The phase variation of the $\sigma \rightarrow \pi\pi$ amplitude is accurately determined as a function of mass from BES II data for $J/\Psi \rightarrow \omega\pi^+\pi^-$. The determination arises from interference with the strong $b_1(1235)\pi$ amplitude. The observed phase variation agrees within errors with that in $\pi\pi$ elastic scattering.

The $\sigma$ pole appears as a conspicuous $\pi^+\pi^-$ peak in BES II data for $J/\Psi \rightarrow \omega\pi^+\pi^-$ [1]. This peak is absent from data on $\pi\pi$ S-wave elastic scattering. The connection between these two processes is a question which is explored here.

For both processes, the partial wave amplitude $f(s)$ may be written

$$f(s) = N(s)/D(s),$$

where $N(s)$ has only left-hand cuts and $D(s)$ has only right-hand cuts. The $N$ function can be different for the two processes. We pursue the hypothesis that $N(s)$ for $\pi\pi$ elastic scattering contains an Adler zero, which is absent from the production process. The phase variation above the $\pi\pi$ threshold arises from the right-hand cut. The $D$ function should be the same for all processes if only a single resonance contributes. The question is whether BES data and $\pi\pi$ elastic scattering data are consistent with this hypothesis.

The Dalitz plot for $J/\Psi \rightarrow \omega\pi^+\pi^-$ is shown in Fig. 1. The $\sigma$ pole appears as a diagonal band at the upper right-hand edge of this plot. There are also strong vertical and horizontal bands due to $b_1(1235) \rightarrow \omega\pi^\pm$. These two bands account for 41% of the data; the $\sigma$ pole accounts for 19% and $f_2(1270)$ for most of the remaining intensity. There is strong interference between the $b_1(1235)$ bands and the $\sigma$ amplitude; this interference provides an accurate determination of the phase $\delta_\sigma$ of the $\sigma$ as a function of $\pi\pi$ mass.

The polarisation of the $\omega$ is along the normal to its decay plane. The $f_2(1270)$ components in the data have angular correlations with this normal which are distinctively different from those of the $\sigma$; as a result, $f_2$ and $\sigma$ are well separated in the mass range where the $\sigma$ amplitude is sizable, up to 1000 MeV. Above this mass, the $\sigma$ amplitude is swamped by the $f_2(1270)$ peak.

The amplitude analysis follows the conventional isobar model. The amplitude for the $b_1(1235)\pi$ final state is parametrised as $\exp(i\Delta_{b_1})F(b_1)$ and that for the $\omega$ is parametrised as $\exp(i\Delta_\omega)F(\sigma \rightarrow \pi\pi)$. Here $\Delta_{b_1}$ and $\Delta_\omega$ are constants describing the strong interaction phases of the 3-body final states $b_1\pi$ and $\omega\sigma$. The $F(b_1)$ amplitude is a Breit-Wigner amplitude of constant width for $b_1(1235)$. A detail is that both S and D-wave decays of $b_1 \rightarrow \omega\pi$ are included, and the D/S ratio of amplitudes is fixed to the PDG value of 0.29 [5].

The $F(\sigma \rightarrow \pi\pi)$ amplitude is taken as [3]:

$$F(\sigma \rightarrow \pi\pi) = \frac{G_\sigma}{M^2 - s - iM\Gamma_{tot}(s)},$$

$$\Gamma_{tot}(s) = g_1 \frac{\rho_{\pi\pi}(s)}{\rho_{\pi\pi}(M^2)} + g_2 \frac{\rho_{\pi\pi}(s)}{\rho_{\pi\pi}(M^2)},$$

$$g_1 = b_1 + b_2 s \frac{s - m_\pi^2/2}{M^2 - m_\pi^2/2} \exp[-(s - M^2)/a].$$

---

*a e-mail: D.Bugg@rl.ac.uk*
Here \( \rho_{\pi\pi} \) is the usual \( \pi\pi \) phase space \( 2k/\sqrt{s} \) and \( k \) is the momentum in the \( \pi\pi \) rest frame. This formalism includes the Adler zero explicitly into \( \Gamma(s) \); the exponential factor cuts off the width at large \( s \). This formula has been fitted simultaneously to BES data [1], CERN-Munich data [4] and the \( K\pi\pi \) data of Pisluk et al. [5]. Our objective is to determine the phase

\[
\delta_\sigma(s) = \tan^{-1}\left( \frac{M\Gamma(s)}{M_s^2 - s} \right).
\]

A small detail is that (2) should strictly contain a dispersive correction to the real part of the amplitude. However, over the mass range covered here, this correction is very small because the phase rises almost linearly with \( s \). The term \( b_1 + b_2 s \) fitted to the data accommodates this small correction.

Another technical detail is that there are actually two \( J/\Psi \to \omega\sigma \) amplitudes having orbital angular momenta \( L = 0 \) and 2 in the production process. These are both included in the fit, with different coupling constants and different strong interaction phases \( \Delta_\omega \). A centrifugal barrier for production with \( L = 2 \) is included, but has little effect since the momentum in the \( \omega\sigma \) final state is large. Likewise, \( L = 0 \) and 2 are both possible for \( J/\Psi \to b_1(1235)\pi \); in practice the \( L = 2 \) amplitude is small.

In the fitting procedure, all amplitudes except that for \( \omega\sigma \) are fitted to the whole data set. In order to determine the phase variation of the \( \sigma \) amplitude with mass, slices 100 MeV wide are examined from \( M_{\pi\pi} = 400 \) to 1000 MeV. Lower masses are not accessible because the \( b_1 \) band runs off the corner of the Dalitz plot; as a reminder, \( s_{\pi\pi} \) varies linearly as one moves perpendicular to the \( \sigma \) band, with the result that low masses are compressed tightly towards the edge of the Dalitz plot.

The determination of \( \delta_\sigma \) has been done in four ways with progressively increasing freedom in the fit, in order to check for consistency. Results are shown as points with errors in panels a–d of Fig. 2. In the first (most restrictive) approach a, only one bin of \( \pi\pi \) mass is examined at a time. The \( \sigma \) amplitude is fitted to the whole \( \pi\pi \) mass range, but allowing a perturbation to the phase \( \delta_\sigma \) of the Breit-Wigner amplitude in a single bin. In the second approach b, both magnitude and phase of \( F(\sigma \to \pi\pi) \) are set free in one bin at a time. In Fig. 2c, the phase is set free in all bins simultaneously, but the magnitude is fitted to the whole mass range in accordance with (2)–(4). In Fig. 2d, the magnitude and phase are fitted freely in all bins simultaneously. Coupling constants of all other amplitudes are re-optimised for every fit.

The full curve of Fig. 2a shows the optimum fit to the whole mass range using (2)–(4). The strong interaction phase difference \( \Delta_{b_1} - \Delta_\sigma \) produces an offset, which is furthermore different for \( L = 0 \) and \( L = 2 \) amplitudes; only the phase variation with mass is meaningful. The curve is therefore drawn so that \( \delta_\sigma = 0 \) at the \( \pi\pi \) threshold. It turns out that the phases \( \Delta_{b_1} \) and \( \Delta_\sigma \) are such that the \( \omega\sigma \) and \( b_1\pi \) amplitudes differ by 90° in phase at \( \pi\pi \) mass of 600 MeV. The interference term between the two amplitudes depends on the cosine of the phase difference, and is therefore determined with maximum sensitivity at this mass.