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Study of the neutron quantum states in the gravity field

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Abstract. We have studied neutron quantum states in the potential well formed by the earth's gravitational field and a horizontal mirror. The estimated characteristic sizes of the neutron wave functions in the two lowest quantum states correspond to expectations with an experimental accuracy. A position-sensitive neutron detector with an extra-high spatial resolution of $\sim 2\,\mu$m was developed and tested for this particular experiment, to be used to measure the spatial density distribution in a standing neutron wave above a mirror for a set of some of the lowest quantum states. The present experiment can be used to set an upper limit for an additional short-range fundamental force. We studied methodological uncertainties as well as the feasibility of improving further the accuracy of this experiment.

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1 Introduction

The quantum states of a particle with mass m in the earth's gravitational field with acceleration g above an ideal horizontal mirror are described by the Schrödinger equation that is analytically solved in textbooks on quantum mechanics [1–5]. One possibility for measuring such states for neutrons has been discussed in [6]. The lowest quantum state of neutrons in such a system was observed in a recent experiment at the Institut Laue-Langevin [7, 8].

The present work aims to confirm the existence of this phenomenon and to study it in more detail. Both the experimental installation and the method of measurement remain similar to those used in our previous experiment. However, the instrumental resolution has been improved by improving scatterer positioning. Statistical accuracy has also been improved by optimising neutron transport in front of the experimental installation as well as by more efficient use of the beam time thanks to the complete automatisation of this measurement. The methodological uncertainties of this method as well as possibilities for reducing them were investigated.

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This article is organized as follows: Section 2.1 describes new features of the experimental installation compared to that used in [8]. Section 2.2 presents the spectral measurement of the horizontal velocity components. The quality of the mirrors used is studied in Sect. 2.3. Section 2.4 considers the factors responsible for the spatial resolution of our method of measuring the neutron quantum states as the neutrons are transmitted through a slit between the mirror and the scatterer. Section 3.1 presents the measurements with different horizontal velocity components and with different scatterers. Section 3.2 deals with the principles of neutron losses in scatterers/absorbers. Agreement between the experimental data and the model is discussed in Sect. 3.3. Section 3.4 deals with the measurement performed with two subsequently installed scatterers – the first to shape the neutron spectrum (it selects 3–4 lower quantum states), the second to analyze the resulting spectral distribution. In Sect. 3.5, we discuss the upper limit for an additional short-range force resulting from this experiment. Finally, Sect. 4 presents the results of a direct measurement of the probability density to observe neutrons as a function of height above mirror. This measurement was carried out using a position-sensitive neutron detector of extra high spatial resolution of $\sim 2\,\mu$m.

The precise analytical solution for the corresponding Schrödinger equation contains Airy-functions. A more transparent and simple solution could be obtained in the quasi-classical approximation [1–3], which is valid for the given potential with a high degree of accuracy of ∼1% even for the lowest quantum state. Thus, in accordance with the Bohr-Sommerfeld formula, the neutron energy in quantum states $E_n (n = 1, 2, 3...)$ is equal to:

$$E_n \cong \sqrt[3]{\left(\frac{9 \cdot m}{8}\right) \cdot \left(\pi \cdot \hbar \cdot g \cdot \left(n - \frac{1}{4}\right)\right)^2} . \quad (1)$$

The precise expression for the energy values E_n has the same property as (1): it depends only on m, g, the Planck constant $\hbar = 6.6 \cdot 10^{-16}\,\mathrm{eV} \cdot \mathrm{s}$ and the quantum number n. On the other hand, a mirror can be approximated to an infinitely high and sharp potential step compared to other characteristic parameters of the problem. Note that the neutron energy in the lowest quantum state (1) $\sim 10^{-12}\,\mathrm{eV}$ is much lower than the effective potential of a mirror $\sim 10^{-7}\,\mathrm{eV}$, and the range of increase of this effective potential $\sim 1\,\mathrm{nm}$ is much shorter than the neutron wavelength in the lowest quantum state $\sim 10\,\mu\mathrm{m}$.

In classical mechanics, a neutron with energy E_n in a gravitational field can rise to the maximum height of $z_n = E_n/mg$. In quantum mechanics, the probability of observing a neutron in an n-th quantum state with an energy E_n at a height z equals to the square of the modulus of its wave function $|\psi_n(z)|^2$ in this quantum state (the precise wave functions can be found in [1–5] or in [7, 8]). Formally, this value is not equal to zero at any height $z > 0$. However, as soon as a height z exceeds some critical value z_n, specific for every n-th quantum state and equals the height of the neutron classical turning point, the probability of observing a neutron approaches zero exponentially fast. This purely quantum effect of neutron penetration into a classically not-allowed region is called the tunneling effect.

An asymptotic expression for the neutron wave functions $\psi_n(z)$ at large heights $z > z_n$ [3] is:

$$\psi_n(\xi_n(z)) \to C_n \cdot \xi_n^{-\frac{1}{4}} \cdot \exp\left[-\frac{2}{3} \cdot \xi_n^{\frac{3}{2}}\right] , \quad \text{if} \quad \xi_n \to \infty , \quad (2)$$

where C_n are known normalization constants;

$$\xi_n = \frac{z}{z_0} - \lambda_n ; \quad (3)$$

and z_0 is the characteristic scale for the gravitational quantum states, which is equal to

$$z_0 = \sqrt[3]{\frac{\hbar^2}{2 \cdot m^2 \cdot g}} . \quad (4)$$

For neutrons at the Earth's surface this value is equal to $5.87\,\mu\mathrm{m}$.

The values λ_n (zeros of the Airy-function) define the quantum energies $E_n = mgz_0\lambda_n$. For the 7 lowest quantum states they are equal to:

$$\lambda_n : \{2.34, 4.09, 5.52, 6.79, 7.94, 9.02, 10.04 \ldots\} . \quad (5)$$

The corresponding classically allowed heights are:

$$z_n = E_n/mg = \lambda_n \cdot z_0 . \quad (6)$$

As soon as such a height z_n is reached, the neutron wave function $\psi_n(z)$ starts approaching zero exponentially fast. For the 7 lowest quantum states they are equal to:

$$z_n = \{13.7, 24.0, 32.4, 39.9, 46.6, 53.0, 58.9 \ldots\}\,\mu\mathrm{m} . \quad (7)$$

Such a wave-function shape has allowed us to define a method for observing neutron quantum states: we measure their transmission through the narrow slit Δz created between a horizontal mirror below and a scatterer/absorber above (from here on generally referred to simplify as a scatterer). If a scatterer is much higher than the turning point for the corresponding quantum state $\Delta z \gg z_n$, the neutrons pass through the slit without significant losses. As the slit size decreases the scatterer starts approaching the neutron wave function $\psi_n(z)$ and the probability of neutron loss increases. If the slit size is smaller than the characteristic size of the neutron wave function in the lowest quantum state z_1, the slit will not be transparent for neutrons. It was this phenomenon that was measured in our previous experiment [7, 8].

2 Method of the measurement

2.1 Experimental installation

The experimental installation and the method of measurement were analogous to those used in our previous work [7–9]. We will therefore just note (where appropriate) the new features; a detailed description of the standard measuring procedure and that of the experimental installation can be found in our previous publications.

A schematic view of the experimental setup is presented in Fig. 1.

The experiment consists of the measurement of the neutron flux through a slit between a mirror and a scatterer as a function of the slit size. Slit size could be finely adjusted and precisely measured. The neutron flux in front of the experimental installation (in Fig. 1 on the left) is

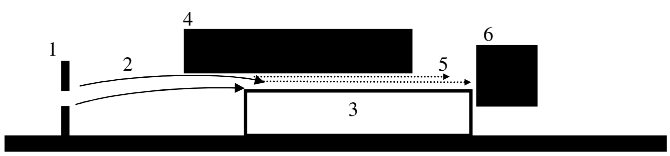

Fig. 1. Schematic view of the present experiment. From left to the right: the vertical bold lines indicate the upper and lower plates of the input collimator (1); the solid arrows correspond to classical neutron trajectories (2) between the input collimator and the entry slit between a mirror (3, empty rectangle below) and a scatterer (4, black rectangle above). The dotted horizontal arrows illustrate the quantum motion of neutrons above a mirror (5), and the black box represents the neutron detector (6). The size of the slit between the mirror and scatterer could be changed and measured