Collisional deexcitation of exotic hydrogen atoms in highly excited states

II. Cascade calculations

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Abstract. The atomic cascades in $\mu^{-}p$ and $\mu^{+}p$ atoms have been studied in detail using new results for the cross-sections of the scattering of highly excited exotic atoms from molecular hydrogen. The cascade calculations have been done with an updated version of the extended standard cascade model that computes the evolution in the kinetic energy from the beginning of the cascade. The resulting X-ray yields, kinetic energy distributions, and cascade times are compared with the experimental data.

PACS. 36.10.-k Exotic atoms and molecules (containing mesons, muons, and other unusual particles)

1 Introduction

The standard cascade model (SCM) of exotic hydrogen atoms, originally introduced by Leon and Bethe [1] and later extended to include the evolution of the kinetic energy distribution during the atomic cascade [2,3], provides a fair description of many properties of atomic cascades, such as the X-ray yields and absorption fractions [4–7]. However, no detailed calculations of the initial stage of the cascade were done until recently due to the lack of the corresponding cross-sections. The upper stage of the cascade was commonly described with a phenomenological deexcitation mechanism, the so-called chemical deexcitation [1]. This mechanism was introduced in order to explain the data on cascade times, as the external Auger effect was found to be too slow at the initial stage of the cascade. The exact nature of the collisional deexcitation of highly excited exotic atoms remained a mystery for a long time until experimental studies of the initial stages of the atomic cascade became possible. Recently the energy distributions of $\pi^{-}p$, $\mu^{-}p$, and $\mu^{+}d$ atoms were studied with various time-of-flight methods [8–12]. In particular, the measurements performed with $\mu^{-}p$ and $\mu^{+}p$ at low density allow one to probe collisional deexcitation mechanisms for highly excited states as long as the energy distribution remains frozen during the lower cascade stage dominated by the radiative transitions.

In our recent paper [13], we studied the dynamics of collisional deexcitation of highly excited exotic atoms using the classical-trajectory Monte Carlo method. The Coulomb transitions with large change of principal quantum number $n$ were found to be the dominant collisional deexcitation mechanism at high $n$, with the molecular structure of the hydrogen target being essential for the dominance of transitions with large $\Delta n$. The main goal of this paper is to investigate the atomic cascades in hydrogen-like atoms at low density and to confront the theoretical results with the experimental data on the X-ray yields, cascade times, and kinetic energy distributions for muonic and antiprotonic hydrogen. As the lower part of the atomic cascade at low target densities is mainly dominated by the radiative deexcitation, the results of our calculations are only weakly affected by theoretical uncertainties in the collisional cross-sections for the low $n$ states. The detailed cascade calculations for high density targets will be published elsewhere.

The article is organized as follows. The cascade model is described in Section 2. The results of the cascade calculations for the $\mu^{-}p$ and $\mu^{+}p$ atoms are presented in Section 3. The conclusions are summarized in Section 4.

2 The extended standard cascade model

2.1 Overview

The extended standard cascade model (ESCM) is a kinetics model that includes all cascade processes of the standard cascade model [1,4,7]: the radiative, Auger, and Coulomb deexcitation, Stark mixing, and, in case of hadronic atoms, nuclear absorption (see Tab. 1 and references therein). In addition, the ESCM also takes into...
account the interaction between internal and external degrees of freedom of the exotic atom: the kinetic energy distribution changes during the cascade due to the acceleration and deceleration mechanisms \([2,3,14,15]\). The new results for the collisional processes presented in \([13,16]\) are used in the present version of the ESCM, a significant improvement over the previous calculations.

The cascade in exotic atoms is divided into two parts: the classical domain for high \(n\) and the quantum mechanical domain for low \(n\). The \(x^−p\) can be described classically for quantum numbers \(n \gg 1\), and for convenience we use the properties of the Auger deexcitation to define the \(n\)-ranges of the two domains. The Auger deexcitation rates are known to have a maximum for \(n = n_c\) where \(n_c\) is the largest \(n\) for which the binding energy released in a \(2n = 1\) transition is enough to ionize the \(H_2\) molecule \([1,13,19]\): \(n_c = 7\) for \(μ−p\) and \(n_c = 12\) for \(pp\). Our calculations show that the Coulomb deexcitation dominates the Auger deexcitation for \(n > n_c\) \([13]\). We define the classical domain by the conditions \(n > n_c\). The classical domain, where the processes were calculated in the classical Monte Carlo (CMC) model with the molecular structure of the target taken into account, will be discussed in Section 2.2. In the quantum mechanical domain, \(n \leq n_c\), the close-coupling method and semiclassical approximations \([16,17,21]\) were used to calculate the differential cross-sections, \(dσ/dcosθ\), of the collisional processes

\[
(x^−p)_{nl} + H \rightarrow (x^−p)_{n'lt} + H 
\]

and the cross-sections for absorption during collision and Auger transitions

\[
(x^−p)_{nl} + H \rightarrow \text{absorption,}
\]

\[
(x^−p)_{n'lt} + H \rightarrow (x^−p)_{nl} + p + e^−
\]

in an energy range relevant for atomic cascade. The new calculations allow us to describe the competition between deceleration and Stark mixing, as well as the absorption in hadronic atoms due to Stark collisions, without employing any fitting parameters related to the Stark mixing and deceleration, like \(k_{Stk}\) used in many earlier calculations \([4]\).

Our cascade code makes full use of the differential cross-sections so that the kinetics is treated more accurately than in cascade models using continuous deceleration \([22]\).

The differential cross-sections used in the present cascade model in the case of muonic hydrogen were calculated in the fully quantum mechanical close-coupling framework for \(n = 2−5\) and in the semiclassical approximation for \(n = 6−7\). The statistically weighted differential cross-sections were used for \(n \geq 6\). In antiprotonic hydrogen, the fully quantum mechanical results are not yet available below the \(ns\) thresholds. Therefore, we used the results of the semiclassical model in the range \(n = 2−9\) and the fixed field model for \(n = 10−12\).

Another approximation used in this paper is related to the Auger deexcitation because the eikonal approximation \([13]\) does not give the differential cross-section and the distribution over final \(l\). Here we use the differential cross-section for the \(l\)-average Stark and elastic transitions and a statistical distribution over final \(l\).

Concerning the acceleration mechanisms, the present calculations include Coulomb deexcitation through the whole cascade. For high \(n\) states, the classical-trajectory Monte Carlo method \([13]\) was used to obtain the cross-sections of the inelastic collisions

\[
(x^−p)_{nl} + H_2 \rightarrow (x^−p)_{n'l} + X, X = H_2, H_2^+, H + H. \quad (4)
\]

For low \(n\), where the classical-trajectory method cannot be expected to give reliable results, we parameterize the Coulomb deexcitation cross-section as follows \(^4\)

\[
σ_{n→n−1}(E_{cm}) = c \frac{n^γ μ_{μp}}{E_{cm} μ_{xp}} \quad (5)
\]

where \(E_{cm}\) is the CMS kinetic energy and \(μ_{μp}\) and \(μ_{xp}\) are the reduced masses of the \(μ−p\) and the \(x^−p\), respectively. We use the values

\[
c = 1.2 \times 10^{−3} a_0^2 \text{ eV}, \quad (6)
\]

\[
γ = 3.5 \quad (7)
\]

which gives a fair description of the Coulomb cross-sections of Bracci and Fiorentini \([23]\) for \(n \leq 7\).

Other calculations \([24−26]\) predict significantly smaller Coulomb deexcitation rates than \([23]\). However, the rapid increase in the Coulomb cross-sections for energies approaching zero is predicted by all models. The dependence on the reduced mass is based on an estimate in \([23]\) and

\[^4\] The \(n = 2\) state in muonic hydrogen is, however, treated differently. Coulomb deexcitation is assumed only to take place in the \(2s\) state and only when the kinetic energy is below the \(2p\) threshold. In this case we use the measured Coulomb deexcitation rate \(λ_{μ^−p→2s}^{\text{Coulomb}} = 4.4 \times 10^{11} N m^{-1}\) \([12]\) where \(N\) is the density of the target in units of liquid hydrogen density (LHD).

### Table 1. Processes included in the extended standard cascade model.

<table>
<thead>
<tr>
<th>Process</th>
<th>Example</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stark mixing</td>
<td>((x^−p)<em>{nl} + H_2 \rightarrow (x^−p)</em>{n'l} + H_2)</td>
<td>([13,16−18])</td>
</tr>
<tr>
<td>External Auger effect</td>
<td>((x^−p)<em>{nl} + H \rightarrow (x^−p)</em>{n'l} + p + e^-)</td>
<td>([13,19])</td>
</tr>
<tr>
<td>Coulomb deexcitation</td>
<td>((x^−p)<em>{nl} + H_2 \rightarrow (x^−p)</em>{n'l} + H_2), (n_l &lt; n_c)</td>
<td>([13,18])</td>
</tr>
<tr>
<td>Elastic</td>
<td>((x^−p)<em>{nl} + H_2 \rightarrow (x^−p)</em>{n'l} + H_2)</td>
<td>([13,16])</td>
</tr>
<tr>
<td>Absorption</td>
<td>((π^−p)_{nl} + H \rightarrow π^0 + n + H)</td>
<td>([16−18])</td>
</tr>
<tr>
<td>Radiative</td>
<td>((x^−p)<em>{nl} + H \rightarrow (x^−p)</em>{n'l} + γ)</td>
<td>([20])</td>
</tr>
<tr>
<td>Nuclear reaction</td>
<td>((π^−p)_{ns} \rightarrow π^0 + n, γ + n)</td>
<td>([1])</td>
</tr>
<tr>
<td>Weak decay</td>
<td>(π^− \rightarrow μ^-ν_μ)</td>
<td></td>
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</tbody>
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