Atom interferometers manipulated through the toroidal trap realized by the interference patterns of Laguerre-Gaussian beams

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Abstract. In this paper, we proposed new constructions of atom interferometers manipulated through the toroidal trap formed by the interference patterns of two co-propagation Laguerre-Gaussian (LG) beams. The coherent splitting and merging of the atomic ensemble, which is essential for the atom interferometer, is realized by the interference pattern of two LG beams. Along the beam propagation direction, a single-well trap is evolved into a double-well trap and then recombined back into a single-well trap, which can be used to form an atom interferometer.

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1 Introduction

In recent years, high-order Laguerre–Gaussian beam received more and more attentions, because the LG beams carry orbital angular momentum [1–3], and can be used to rotate micro-particles [4], the Bose-Einstein condensation [5] and the atom cloud [6]. There are many methods to produce the LG beams, such as the mode conversion method with cylindrical lens [7], the spiral phase-plate method [8], the micro-imaging method for hollow fiber modes [9], the holograms method [10], etc. The LG beams have the hollow light modes, which can be used to store atom in the dark area of the beam for blue-detuned atoms [11,12]. Moreover, The LG beams can produce toroidal confinement in the bright area of beam for red-detuned atoms to form the vortex state of atoms [13–15]. A toroidal geometry can enable studies of phenomena in non-simply connected or low-dimensional topologies [16–19], e.g., super-fluid persistent circulation states of Bose-Einstein condensates (BEC) [16].

It is an effective technique to use the interference pattern of optical beams to produce and manipulate the ensembles of cold atoms. For instance, the coherence of BEC was studied by suddenly releasing the atoms from the lattice potential, which is formed by the interference of two counter-propagating Gaussian beams. In this way, the multiple matter-wave interference fringes of several thousand expanding quantum gases were observed [20]. Recently, the optical dipole traps and fractional Talbot optical lattices based on the interference between multiple co-propagating Gaussian beams, were used to illustrate the spatial translation and splitting of atoms among a set of the interference traps [21]. A weak standing-wave hollow-beam gravito-optical trap composed of the interference of two counter-propagating doughnut hollow beams and a plug beam is proposed, which can be used to realize the intensity-gradient induced Sisyphus cooling of neutral atoms [22]. Also, a scheme for the generation of arbitrary coherent superposition of vortex states in BEC using the interference of the LG beams by Mach-Zender type configuration is proposed [23]. In 2007, Franke-Arnold et al. obtained both bright and dark optical ring lattices from pairs of Laguerre-Gaussian modes with different azimuthal orders. Furthermore they have shown that, in combination with a magnetic trap, these lattices will be ideal for studying quantum degenerate gases [24].

There are several methods to construct the atom interferometer. One way is using mechanical splitter to split the atom beam, such as Young’s double-slit atom interferometer [25] and three-grating atom interferometer [26]. Another way is using magnetic field of current carrying wires with Y-shaped [27] and double-X-junction [28] to split and recombine atoms ensemble, which has the shortage of the shot noise in current carrying wires. The third way is using the optical potential. Based on the adiabatic transformation of a single optical trap into a double well trap and then recombined back into a single optical trap, the atom interferometer can be formed, which were actively investigated [21,29–31].

In this paper, we will show how to use the characteristics of the LG beams and the interference patterns of two co-propagation LG beams to form atom interferometers. The construction of atom interferometers by using
the toroidal optical trap is analyzed. Along the beams propagation direction, due to the coherent superposition of two LG beams, a single well trap is evolved into a double well trap and then recombined back into a single well trap, which can be used to form an atom interferometer. The depth of dipole trap and the minimum time between the photons scattering events for Rb$^{87}$ atom interferometers are also calculated.

2 The characteristics of the interference of two co-propagation Laguerre-Gaussian beams

The electric field of Laguerre-Gaussian beam can be expressed as following [1]:

$$E(r, \phi, z, k) = A(r, z) \exp[-i\psi(r, \phi, z, k)],$$  \hspace{1cm} (1)

$$A(r, z) = G_0 \frac{1}{w_z} \left(\frac{\sqrt{2r}}{w_z}\right)^l L_m^l \left(\frac{2r^2}{w_z^2}\right) \exp\left(-\frac{r^2}{w_z^2}\right),$$  \hspace{1cm} (2)

$$\psi(r, \phi, z, k) = k z + \frac{k r^2}{2} \frac{z}{z^2 + z_0^2} - (2m + |l| + 1) \text{tg}^{-1} \left(\frac{z}{z_0}\right) + l \phi,$$  \hspace{1cm} (3)

where $G_0 = \sqrt{2P_0 m! \pi (m + l)!}$ is the normalization constant to make the light power to be $P_0$, $w_z = w_0 \sqrt{1 + z^2 / z_0^2}$ is the radius of beam, $w_0$ is the waist radius, $z_0 = w_0^2 / \lambda$ is the Rayleigh range, $k = 2\pi / \lambda$ is the wave number. $L_m^l(2r^2 / w_z^2)$ is the Laguerre polynomial, $l$ and $m$ are the azimuthal and radial order of beam respectively. $l$ refers to the number of $2\pi$ phase cycles around the circumference of the mode and $(m + 1)$ indicates the number of radius nodes in the mode profile.

In the following, we consider LG beams with radial order $m = 0$. The mode profile of LG beam with azimuthal order $l = 0$ is Gaussian. For different value of azimuthal order $l \neq 0$, the profile of the light mode is hollow Gaussian, which means the intensity in the center of the mode profile is zero, and has its maximum value at position $r = r_{max} = \sqrt{l / 2w_z}$, as depicted in Figure 1. For the red-detuning atoms, which approach high intensity in the center of the interference patterns, the Gouy phase changes resulting in a modification of the interference pattern.

Let us consider the interference of two co-propagating LG beams with the same value for azimuthal order $l$ and radial order $m = 0$ along the $z$ directions, whose beam waist centers are located at the positions $(0, \pm d/2, 0)$ respectively, as shown in Figure 1. $d$ is the separation between the two beam waist centers in the transverse plane. This system is similar to Young’s double-slit geometry. Based in equations (1)–(3), the intensity distribution of the combined beam can be obtained as follows:

$$\frac{I}{I_0} = \frac{1}{w_z^2} \left[ \frac{2x^2 + 2(y + d/2)^2}{w_z^2} \right]^{\frac{|l|}{2}} \exp\left(\frac{2x^2 + 2(y + d/2)^2}{w_z^2}\right) + \frac{1}{w_z^2} \left[ \frac{2x^2 + 2(y - d/2)^2}{w_z^2} \right]^{\frac{|l|}{2}} \exp\left(\frac{2x^2 + 2(y - d/2)^2}{w_z^2}\right) + 2 \frac{\sqrt{2\pi} (2l^2 / w_z^2)}{w_z^2} \sqrt{x^2 + (y - d/2)^2} \exp\left(\frac{2\sqrt{x^2 + (y + d/2)^2}}{w_z^2}\right) \left[ \frac{2\sqrt{x^2 + (y + d/2)^2}}{w_z^2} \right]^{\frac{|l|}{2}} \times \exp\left(\frac{2\sqrt{x^2 + (y + d/2)^2}}{w_z^2}\right) \cos\left(\frac{kd}{\lambda (z^2 + z_0^2)}\right) \left( \frac{y + d/2}{x} \right) - \cos\left(\frac{y - d/2}{x}\right) + \Delta \varphi,$$  \hspace{1cm} (4)

where $\Delta \varphi$ is the difference of original phase between two beams. From equation (4), we can see that each individual beam is a hollow tube of bright light. Away from the beam focus, the Gouy phase changes resulting in a modification of the interference pattern.

According to equation (4), the characterization of interference patterns of two LG beams can be analyzed. At the center of the interference patterns $x = y = 0$, the cosine term in equation (4) becomes $\cos[-k\pi + \Delta \varphi]$. In order to obtain the maximum intensity at the center of the interference patterns, the phase difference should be $\Delta \varphi = \pi$ for the LG beam with odd azimuthal order, and it should be $\Delta \varphi = 0$ for the LG beam with even azimuthal order. For the central interference patterns $x = 0$ and $y = 0$, the maximum intensity of central pattern is located at $z = z_{max} = z_0 \sqrt{d^2 / (2l^2 + 2)w_0^2} - 1$ with intensity

![Fig. 1. The schematics of two LG beams interference constitution with two waists located at position $(0, \pm d/2, 0)$.](image-url)