Load-dependent random walks on complex networks

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Abstract. Load-dependent random walks are used to investigate the evolution of load distribution in transportation network systems. The walkers hop to a node according to node load of the last time step. The preference of walks leads to a change in the load distribution. It changes from degree-dependent distribution in the case of non-preference walks to eigenvector-centrality-dependent distribution. By numerical simulations, it is shown that the network heterogeneity has a influence on the effect of walk preference. In the cascading failure phenomenon, an appropriate degree correlation can guarantee a low risk of cascading failures.

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1 Introduction

Recently, much attention has been paid on the study of the structural properties of complex networks due to the discovery of the prevalence of scale-free (SF) nature in most real-world networks [1] and the great influence of the network structure on the dynamical processes of the networks [2–4]. The analysis of percolation-based models reveals that scale-free networks are stable against random removal of nodes, while they are fragile under intentional attacks targeting on nodes with high degree [5–9]. Besides static failures, dynamical cascading failure phenomenon has also been studied. A small amount of nodes fails initially (usually called initial failure). Consequently these nodes and the related links are removed from the network. The removal leads to redistribution of the flow. After redistribution, some of the remaining nodes and links are loaded with a larger flow than before and probably fail in case of overload (load exceeding load capacity), giving rise to more flow redistribution and possibly more failure. Eventually a network-wide cascading failure may occur [10–17].

Random walks are probably the simplest stochastic process affected by the network topology. Random walks have been studied in different types of networks [18–21]. In random walks, a walker at a node (site) \( i \) and time \( t \) selects one of its \( K_i \) neighbors with equal probability to which it hops at time \( t + 1 \). However, the transition probabilities are not always equal. Sometimes a part of links is relatively busy while scarcely goes through other links. Walkers favor some links sometimes only because walkers have met through the links before. Therefore these links are more widely known. Though such preference mechanism was studied in network construction, it needs to be further investigated in complex networks [18].

Most studies of cascading failures use betweenness centrality (BC) to represent loads on nodes or links [12–16]. Such representation is based on an assumption that each pair of nodes in a network transmit the same amount of loads, for example, electrical power or communication packers. However actual loads transmitted between each pair of nodes are not necessarily the same. Most studies of cascading failures often randomly selected some initially failed nodes. Some studies simulated intentional attacks when large-degree nodes were deliberately selected to be initially failure. However, if the initial failures arise from the evolution of load distribution, some network components may suffer overload ahead of other components from their local connection conditions. These overload failures may trigger cascading failure. Thus, the cascading failures with initial failures arising from the load redistributions need to be further investigated.

In this paper, the preference-random-walks model is investigated. In Section 2, a load-dependent random walk model is introduced to study the evolution of load distribution. In Section 3, numerical simulations of load-dependent walks are performed and the results are presented. The homogeneous and heterogeneous networks are compared in Section 4. The cascading failures arising from preference-induced evolution of load distribution are numerically simulated in Section 5. An analysis of the influence of degree correlation is presented in Section 6. A discussion and conclusion are presented in Section 7.
2 Load-dependent random walk model

The load-dependent random walks can be considered as follows [22,23]. At each time step, \( M \) walkers move on a network consisting of \( N \) nodes. It is assumed that the network is connected, i.e., there is a path between each pair of nodes. Otherwise each component can be considered separately. The connectivity is characterized by the adjacency matrix whose element \( \eta_{ij} = 1 \) if there is a link between \( i \) and \( j \), and \( \eta_{ij} = 0 \) otherwise. An undirected network is restricted, namely \( \eta_{ij} = \eta_{ji} \) and hops in two directions are allowed. The degree that the number of links node \( i \) has is denoted by \( K_i \). At each time step, there are \( M \) walkers hoping a step, from a node (source node) to one of the nearest neighboring nodes (destination node). At the next time step, other \( M \) walkers hop. That is to say, the walks are not continuous. The walks simulate the traffic manner that different walkers (or traffic flows) move at every time step, rather than a fixed group of walkers always moving unceasingly. Load of node \( i \) at time step \( t \) (\( L_i(t) \)) is defined as the relative number of visitors, namely the ratio of the number of walkers arriving at node \( i \) to the total walkers number \( M \).

It is assumed that the random walks at this time step are strongly dependent on node loads at the previous time step. So the walks considered here are load dependent in this sense. The more walkers arrive at node \( i \) at the previous time step, the more likely to depart from node \( i \) at this time step. Node \( j \) receives walkers from node \( i \) with a probability proportional to load of node \( i \) at \( (t-1) \) time step, namely \( L_i(t-1) \). Therefore the probability \( P_i(t) \) of a walker to depart from node \( i \) to one of \( K_i \) nearest nodes is expressed as \( P_i(t) = \sum_{j=1}^{K_i} \eta_{ij} L_i(t-1) = K_i L_i(t-1) \). This assumption is justified by a simple idea that more visitors arrive at this time step, more will leave at the next time step. If the walkers number is small relative to nodes number, though the probability \( P_i(t) \) is not zero, there may be no walker departing from node \( i \) by accident. To avoid this, the condition \( M \gg N \) is assumed.

3 Load redistribution

The numerical simulations of load-dependent random walks on a real-world network, US-air line network, is plotted in Figure 1. Figure 1e shows the connecting probability \( P(D) \) that a node is connected immediately to other \( K \) nodes (normalized degree \( D_i = K_i / \sum_{i=1}^{N} K_i \)). The probability \( P(D) \) scales as a power-law, \( P(D) \propto D^{-\gamma} \). Therefore this is a typical SF network. Node number \( N = 332 \). The datasets of the networks were collected from Pajek program datasets URL: \text{http://vlado.fmf.uni-lj.si/pub/networks/data/}. At each time step, the walker number is \( M = 10^3 \). In the initial 100 time steps, \( 0 < t \leq 100 \), no preference is applied to random walks. After time \( t = 100 \), the load-dependent preference is switched on. The same initial nodes are chosen for all nodes, i.e., \( L_i(0) = \frac{1}{N} = 3.01 \times 10^{-3}, \forall \). Figure 1a is the phase plot between load \( L_i(t = 100) \) and normalized degree \( D_i \). The plot exhibits a thin straight line, indicating that the load of each node linearly depends on node degree in the case of non-preference walks. This linear dependence can be simply explained as follows. In every time step walkers randomly select links to go through. Thus, the more links a node has, the more probably the node receives walkers. The linear dependence of loads on degrees for non-preference walks has also been reported [22,23].

After time \( t = 100 \), the preference is switched on. The linear dependence of load on degree is lost, as seen in phase plots for \( t = 101, 102, \) and \( 200 \) (Figs. 1b, 1c, and 1d respectively). In the time region \( t \leq 100 \), all nodes weigh equally and source nodes are completely randomly