**Classical radiation in optical fibers as model for quantum entanglement**

**Classical radiation and entanglement**

M.A. Man’ko

P.N. Lebedev Physical Institute, Moscow

**Abstract.** Using known description of the classical electromagnetic radiation propagating in optical fibers by the Schrödinger-like equation (Fock–Leontovich paraxial approximation), the model of entanglement phenomenon is considered. Gaussian light beams in optical fibers demonstrate a formal mathematical identity to Gaussian two-mode quantum fields. In view of this, the criterion of separability and entanglement of Gaussian light beams is implemented in fiber optics.

1 Introduction

Recently it was suggested [1] to use light beams in optical fibers to study different elements of quantum information processing. The idea of the implementation of classical light beams to imitate the quantum information processing is based on a quantumlike behavior of the beams.

The light-beam propagation in optical fibers is described by the Schrödinger-like equation. This equation follows from classical Maxwell’s equations for electromagnetic signals, if one uses the Fock–Leontovich (paraxial) approximation for the light beam in optical fibers (see, e.g., [2–4]). Due to this, the solutions to the Schrödinger-like equation demonstrate quantumlike properties. For example, modes of light beams formally are equivalent to wave functions of a quantumlike system with two degrees of freedom. Thus such quantum aspects of quantum information processing as entanglement of systems with two degrees of freedom can be considered as analogous properties of the light modes propagating in optical fibers.

The aim of this paper is to review the approach to quantum information processing suggested in [1] (see also [5]) based on using a quantumlike behavior of light beams in optical fibers. It is worth noting that the quantumlike methodology was developed by Renato Fedele (see [6] and references therein).

The paper is organized as follows.

A short presentation of the Fock–Leontovich approximation is given in section 2. In section 3 the imitation of uncertainty relations is presented. Modes in optical fibers are considered in section 4 and the beam propagator along the optical-fiber axis is used for simulation of teleportation in section 5. Examples of selfoccs, qubits, and coherent states for the application in quantum information processing are done in section 6. In section 7, imitation of entanglement is proposed, while in section 8 the Kerr-like medium profile of the refractive index is applied for treating the CNOT gate. Gaussian states and ppt-criterion of separability of composed-system state are reviewed in section 9. In section 10 conclusions and perspectives are given.
2 Fock–Leontovich approximation for paraxial beams of electromagnetic radiation

In reality, it is not unusual that purely classical systems can be described by a quantumlike equation. For example, Fock and Leontovich used this ansatz in their study of the electromagnetic wave propagation along the Earth’s surface [7,8]. They have shown that Maxwell’s equations in nondispersive media for paraxial beams of the electromagnetic field can be reduced to a Schrödinger-like equation (the so-called paraxial approximation).

In fact, since it is the standard procedure for studying the electromagnetic waves, starting from Maxwell’s equations, one obtains the Helmholtz equation for the component of the electric field. The Helmholtz equation is obtained for the wave with given frequency, neglecting the media dispersion and influence of polarization,

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial z^2} + k^2 n^2(x, z) E = 0$$ (1)

(we consider a slab or planar waveguide configuration). In Eq. (1), \( \lambda = 2\pi/k \) is the wavelength in vacuum, \( z \) is the longitudinal coordinate, and \( n(x, z) \) is the refractive index.

For paraxial beams, we introduce the complex function \( \psi(x, z) \), which is a slowly varying amplitude of the electric field, using the following formula:

$$E(x, z) = n_0^{-1/2}(z) \psi(x, z) \exp \left[ ik \int_0^z n_0(\xi) d\xi \right].$$ (2)

The ansatz (2) reduces the Helmholtz equation (1) to the Schrödinger-like equation

$$i\lambda \frac{\partial \psi(x, z)}{\partial z} = - \frac{1}{2n_0(z)} \frac{\partial^2 \psi(x, z)}{\partial x^2} + U(x, z) \psi(x, z),$$ (3)

with \( U(x, z) \) being an effective potential related to the refractive index of the medium \( n(x, z) \) as

$$U(x, z) = \frac{1}{2n_0(z)} \left[ n_0^2(z) - n^2(x, z) \right],$$

where \( n_0(z) = n(0, z) \) is the refractive index of the medium at the beam axis. While deriving this equation, one neglects the second-order derivatives of \( \psi \) with respect to the coordinate \( z \) and the derivatives of the function \( n_0(z) \), that can be done in the case of a slow variation of the refractive index along the beam axis:

$$\frac{\lambda}{n_0^2(z)} \left| \frac{dn_0(z)}{dz} \right| \ll 1.$$  

This inequality means that the refractive index has very small changes in the direction of \( z \)-axis on the distance of the order of the wavelength \( \lambda \).

Analogously, for optical waveguides (optical fibers), we obtain that the light beam propagating in the optical fiber is described by the equation

$$i\lambda \frac{\partial \psi(x, y, z)}{\partial z} = - \frac{\lambda^2}{2n_0(z)} \left( \frac{\partial^2 \psi(x, y, z)}{\partial x^2} + \frac{\partial^2 \psi(x, y, z)}{\partial y^2} \right) + \frac{1}{2n_0(z)} \left[ n_0^2(z) - n^2(x, y, z) \right] \psi(x, y, z).$$ (4)

This is a Schrödinger-like equation for a wave-like function of a system with two degrees of freedom (coordinates \( x \) and \( y \)). The longitudinal coordinate \( z \) along the fiber axis plays the role of time in the Schrödinger equation (see Fig. 1). The \( z \)-evolution of the electromagnetic field is described by the evolution operator \( \hat{U}(z) \) as

$$\hat{U}(z) \psi(x, y, 0) = \psi(x, y, z),$$ (5)