Ageing single file motion

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Abstract. The mean squared displacement of a tracer particle in a single file of identical particles with excluded volume interactions shows the famed Harris scaling \( \langle x^2(t) \rangle \approx K_1 t^{1/2} \) as function of time. Here we study what happens to this law when each particle of the single file interacts with the environment such that it is transiently immobilised for times \( \tau \) with a power-law distribution \( \psi(\tau) \approx (\tau^\star)^\alpha \), and different ranges of the exponent \( \alpha \) are considered. We find a dramatic slow-down of the motion of a tracer particle from Harris’ law to an ultraslow, logarithmic time evolution \( \langle x^2(t) \rangle \approx K_0 \log^{1/2}(t) \) when \( 0 < \alpha < 1 \).
In the intermediate case \( 1 < \alpha < 2 \), we observe a power-law form for the mean squared displacement, with a modified scaling exponent as compared to Harris’ law. Once \( \alpha \) is larger than two, the Brownian single file behaviour and thus Harris’ law are restored. We also point out that this process is weakly non-ergodic in the sense that the time and ensemble averaged mean squared displacements are disparate.

1 Introduction

The mean squared displacement of a single tracer particle in a simple liquid is typically of the linear form

\[ \langle x^2(t) \rangle = 2dK_1t, \] (1)

the characteristic scaling for a Brownian particle, where \( d \) is the embedding dimension and \( K_1 \) the diffusion coefficient. For a random walker on a \( d \) dimensional lattice with lattice spacing \( a \), \( K_1 = a^2/[2d\tau_0] \), where \( \tau_0 \) is the typical time for a single jump. In three dimensions, as long as the density of particles is not overly large, the motion of individual particles with excluding volume interactions will still be governed by Brownian motion, as the probability of particle-particle encounters is relatively small. However, when we confine the motion of the particle to one dimension, particles will eventually bump into each other. The resulting many-body interactions severely alter...
the Brownian law. As shown by Harris in 1965, the motion of a tracer particle in such a single file of particles is characterised by the square-root scaling

\[ \langle x^2(t) \rangle \simeq K_{1/2} t^{1/2} \]  

(2)

of the mean squared displacement \[ 1 \]. Here \( K_{1/2} \) is an anomalous diffusion coefficient of dimension cm/sec\(^{1/2} \) and the symbol \( \simeq \) denotes the asymptotic equality up to a constant prefactor. In the scenario discussed below, the Harris result (2) corresponds to a continuous time random walk process with an exponential distribution of waiting times.

Experimentally, single file motion was shown to be realised for colloidal particles moving in circular grooves \[ 2 \], an experiment in which also the asymptotic Gaussian character of the probability density of the particle position was demonstrated. Channels can also be realised by help of optical tweezers \[ 3 \], and the motion of excluded volume particles in such channels exhibits a turnover from initial free diffusion to single file motion with the \( t^{1/2} \) scaling of the mean squared displacement \[ 4 \]. Also in this experiment the Gaussian nature of the propagator was shown \[ 4 \]. Apart from these direct single particle tracking assays, single file diffusion was also demonstrated by pulsed field gradient NMR in zeolite structures, so-called molecular sieves \[ 5 \] and further analysed by simulations \[ 6 \]. Single file diffusion is also characteristic for molecular biological processes, for instance, the transport of biomolecules through cell membranes \[ 7 \]. Moreover, this type of motion is observed in microchannel setups \[ 8 \] as well as in nanochannels \[ 9 \].

Single file diffusion has been studied extensively by analytical and simulations approaches, see, for instance, Refs. \[ 10–14 \] with respect to the influence of various physical parameters such as the density of single file particles. More recently, the field of single file diffusion has received renewed attention. We mention the description of single file diffusion on a finite interval \[ 15 \], the effect of the particle density \[ 16 \], as well as single file motion of externally driven particles \[ 17,18 \]. A major step forward in the understanding of a tagged particle in a single file is the harmonisation approach of Ref. \[ 19 \], which shows that the motion of the particle is described by a fractional Langevin equation.

Here we consider the generalisation of the single file dynamics to a case when the particles in the single file interact with a disordered environment. Consider a single file of functionalised colloidal particles moving in a channel whose surface is functionalised complementarily to the colloidal particles, effecting transient sticking of the colloids to the wall of the channel with a power-law distribution \( \psi(\tau) \sim (\tau^*)^\alpha \). The existence of such power-law distributed sticking times was indeed shown experimentally for colloids, which bind transiently to a wall \[ 20 \]. This scenario gives rise to a logarithmic growth of the mean squared displacement instead of Harris’ law (2).

We stress that the approach taken herein—based on the physical scenario of sticky colloid-wall interactions—is different from that of previous works \[ 21–23 \].

## 2 Ultraslow single file diffusion

Consider first a single particle, which successively becomes immobilised for periods \( \tau \), which are distributed according to the power-law probability density

\[ \psi(\tau) = \frac{\alpha}{\tau^* [1 + \tau/\tau^*]^{1+\alpha}} \simeq \frac{(\tau^*)^\alpha}{\tau^{1+\alpha}}, \]  

(3)

where the scaling exponent \( \alpha > 0 \) and \( \tau^* \) is a microscopic time scale. When \( 0 < \alpha < 1 \) the distribution \( \psi(\tau) \) does not possess a finite characteristic time scale