Relationship between coupler curve properties of a 4-bar linkage and its dimensional types

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Abstract  Three types of 4-bar linkages can trace the same coupler curve according to the cognate mechanism law. Based on the investigation of the relationship between dimensions of a linkage and the harmonic component characteristic parameters of the coupler curve of the linkage, it was found that six types of linkages are related to the same coupler curve, and they belong to two groups of cognate mechanisms, in which the coupler curves have the same shape and are symmetric. The relationships among the dimensional types of these six mechanisms are presented. The corresponding examples have also proved these conclusions. This work enriches the cognate mechanism law, and can result in more candidate mechanisms for mechanism dimensional synthesis.

Keywords: 4-bar linkages, cognate mechanisms, dimensional type.

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That the same coupler curve can be traced by different cognate mechanisms enriches the dimensional synthesis of mechanism and can result in more candidate mechanisms for our design. According to the Roberts-Chebyshev law, that is the cognate mechanism law, one coupler curve can be generated by three mechanisms with different dimensions. Many publications about cognate mechanisms have become classics. Ref. [1] gave the background of the cognate mechanism law and how to get the cognate mechanism dimensions by Cayley Drawing Method; in ref. [2], a different method was proposed for determining the cognate mechanism dimensions by a complex vector. In ref. [3], the cognate mechanism was regarded as one part of the solutions of path and function generators. There are some advantages in cognate mechanisms. For example, when the original mechanism cannot satisfy the design requirement, the cognate mechanisms can be used to replace the original one, since they have different kinematic properties, transmission angle and occupied space. The properties of the transmission angle of cognate mechanisms were also investigated in depth in ref. [4]. Until now, it is not clear what is the inherent law between original mechanism dimensions and the cognate ones, so it is of great significance to find out the inherent relationship among different mechanism dimension types with the same couple curve, and to present a convenient method
for dimensional synthesis.

Based on further investigation of the coupler curve of the cognate mechanism by using the Fast Fourier Transform (FFT), the inherent relationship between the original mechanism dimensions and the cognate ones was found according to the three discoveries. Six mechanisms are found rather than three, which can trace the coupler curve with the same shape, and the six mechanisms belong to two groups of cognate mechanisms. The research in this paper enriches the cognate mechanism law, and can result in more candidate mechanisms for mechanism dimensional synthesis.

1 Basic definitions and corresponding concepts

1.1 Mathematical description of coupler path

In fig. 1, the length of each link of a 4-bar linkage $ABCD$ is noted as $(l_1, l_2, l_3, l_4)$, the position of point $P$ on coupler $BC$ is represented as $r_0 e^{j\alpha_0}$, and $r_B(t)$ stands for the vector of point $B$ in the coordinate. When coupler $BC$ moves with crank $AB$, the path of point $P$ can be described as follows:

$$r_P(t) = r_B(t) + r_0 e^{j(\alpha_0+\alpha(t))} = r_B(t) + r_0 e^{j\alpha_0} \cdot e^{j\alpha(t)},$$

where, $e^{j\alpha(t)}$, defined as the oscillating function of coupler $BC(t)$, and $\alpha(t)$ are just determined by dimensions type of $l_1$, $l_2$, $l_3$ and $l_4$, that is $(a, b, c, d)$, and obviously $l_1 : l_2 : l_3 : l_4 = a : b : c : d$.

1.1.1 Mathematical description of coupler path with standard arrangement. A standard arrangement of a mechanism means that the frame $AD$ lies on $x$-axis; point $A$ is located at the origin of coordinates; initial angle of $AB$ is $0^\circ$ (relative to $x$-axis) (see fig. 2).

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