On Predicting Forming Limits Using Hill’s Yield Criteria

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The analysis of localized necking is strongly dependent on the yield function. To predict forming limits, therefore, numerous yield criteria have been postulated to characterize the plastic deformation of sheet materials. Among them Hill’s 1948 and the fourth form of 1979 yield criteria are the most commonly used. A new yield criterion was proposed by Hill in 1993. It uses five independent and easily obtainable material parameters, which makes it flexible in representing the shape of the yield locus for different materials. The present investigation compares these three yield criteria in forming limit predictions based on both the Marciniak and Kuczynski (M-K) approach and the bifurcation analysis. It is observed that the M-K analysis based on Hill’s 1993 yield criterion provides forming limit predictions in agreement with experimental data. The bifurcation analysis based on Hill’s 1948 yield criterion also provides an acceptable prediction of forming limits for aluminum, although they are slightly higher. All three yield criteria are found to provide acceptable predictions for aluminum-killed (AK) steel based on the M-K method. For brass, only the prediction based on the M-K method and Hill’s 1993 yield criterion is close to the trend of experimental data.

Keywords: plastic deformations, forming limits, aluminum-killed steel, aluminum sheet

1. Introduction

One of the common failure modes in sheet forming is localized necking. Forming limit diagrams (FLDs) are usually used to characterize the formability of sheet metal. Experimental evidence has shown that material properties, sheet thickness, strain paths, and surface finish are the major factors controlling the formability of sheet metals. Marciniak and Kuczynski (M-K)[1] introduced the concept of an initial imperfection in the sheet, which develops into a localized neck when the load applied to the uniform region of the sheet increases and force equilibrium between the groove and the outside region is maintained. The M-K method has been widely used to predict forming limits of various materials.[2-5] Another approach to the analysis of localized necking considers that a bifurcation mode is assumed to indicate the initialization of localized necking. where analysis is based on deformation theory of rigid-plastic material.

The geometric configuration of the yield surface has a significant influence on predicted forming limit strains. Many yield criteria have been proposed to reflect the material properties of sheet metals. Hill’s 1948 yield criterion has been used extensively to predict forming limits of aluminum-killed (AK) steel based on the M-K method.[13-17] However, for aluminum sheet, significant discrepancies exist between experimental data and predictions when this criterion is used. To accommodate the anomalous behavior of aluminum,[18] a second yield criterion was postulated by Hill in 1979.[13] Analysis based on the fourth form of this yield criterion shows an improvement in forming limit predictions for aluminum sheet.[19] In 1993, Hill proposed a new and user-friendly yield criterion.[14] This criterion has five independent material properties. Thus, it may have flexibility in representing the yield locus of various materials. Since Hill’s 1948 and 1979 yield criteria are the most commonly used yield criteria in predicting forming limits, and Hill’s 1993 yield criterion has a potential of wide applications, the question may be which yield criterion should be used under certain circumstances. Therefore, the present investigation is focused on the comparison of the yield criteria proposed by Hill in 1948, 1979, and 1993 and their effect on forming limit predictions. The shape of yield loci of these criteria is discussed. The forming limits predicted using both the M-K approach and the bifurcation analysis are compared for the three yield functions. For selected materials, predicted forming limits are compared with experimental data.

2. Hill’s Yield Criteria

2.1 The 1948 Yield Criterion

The original form of this yield function was given as [12]

\[ 2f = F(s_1 - s_2)^2 + G(s_2 - s_3)^2 + H(s_3 - s_1)^2 + 2L\tau_{\text{xx}}^2 + 2M\tau_{\text{xy}}^2 + 2N\tau_{\text{yy}}^2 = 1 \]  
(Eq 1)

where \( F, G, H, L, M, \) and \( N \) are constants, which describe the characteristics of material anisotropy. These constants can be determined by tensile yield stresses in the principal anisotropic directions and yield stresses in shear. They can also be determined by introducing strain ratios \( r_{\text{xx}}, r_{\text{xy}}, \) and \( r_{\text{yy}} \) for the plane stress condition. Considering sheet metal forming and assuming that the sheet has planar isotropy, Eq 1 reduces to

\[ \sigma_1^2 + \sigma_2^2 + r(\sigma_1 + \sigma_2)^2 = (1 + r)\sigma_0^2 \]  
(Eq 2)

where \( \sigma_0 \) is the in-plane uniaxial tensile stress, \( r \) is the normal anisotropic strain ratio, and \( \sigma_1 \) and \( \sigma_2 \) are principal stresses. The loci of Eq 2 are shown in Fig. 1. They are ellipses with major and minor axes depending on the \( r \) value. A higher value of \( r \) causes a higher value of the yield stress under biaxial tension. For balanced biaxial tension, Eq 2 reduces to...
The present analysis will focus on this equation. Lian et al. pointed out that the yield locus of the fourth equation remains convex as long as the exponent $M$ is greater than unity. The effective strain is given by work equivalence as

$$\varepsilon = \frac{1}{2} \left\{ (1 + r) \right\}^{\frac{1}{M}} \left[ |\varepsilon_1 + \varepsilon_2|^{\frac{M}{M-1}} + (1 + 2r) (|\varepsilon_1 - \varepsilon_2|^{\frac{M}{M-1}}) \right]^{\frac{M-1}{M}}$$

(Eq 6)

It is noted that when $M = 2$, Eq 5 reduces to the 1948 yield function, Eq 2. Substituting the yield stress under balanced biaxial tension $\sigma_r$ into Eq 5 results in

$$\left( \frac{\sigma_r}{\sigma_u} \right)^M = \frac{1 + r}{2^{M-1}}$$

(Eq 7)

Equation 7 indicates that the anomalous behavior exists with $2^{M-1} > 1 + r$ when $r > 1$, or $2^{M-1} < 1 + r$ when $r < 1$. Denoting $\alpha_b = \sigma_r/\sigma_u$, the stress exponent $M$ is obtained from Eq 7:

$$M = \ln(2(1 + r))/\ln(2/\alpha_b)$$

(Eq 8)

### 2.3 The 1993 Yield Criterion

The yield function proposed by Hill in 1993 is

$$\frac{\sigma_x^2}{\sigma_y^2} - c\frac{\sigma_y^2}{\sigma_x\sigma_y} + \frac{\sigma_y^2}{\sigma_x\sigma_y} + \left\{ (p + q) - \frac{p\sigma_1 + q\sigma_2}{\sigma_y^2} \right\} \frac{\sigma_1\sigma_2}{\sigma_x\sigma_y} = 1$$

(Eq 9)

where $c$, $p$, and $q$ are nondimensional parameters given by

$$\frac{c}{\sigma_y^2} = \frac{\frac{1}{\sigma_x^2} + \frac{1}{\sigma_y^2} - \frac{1}{\sigma_b^2}}{\sigma_y^2}$$

(Eq 10)

$$\left( \frac{1}{\sigma_x^2} + \frac{1}{\sigma_y^2} - \frac{1}{\sigma_b^2} \right) p = \frac{2r_b(\sigma_b - \sigma_y)}{(1 + r_b)\sigma_y^2} - \frac{2r_b\sigma_y}{(1 + r_b)\sigma_y^2} + c$$

$$\left( \frac{1}{\sigma_x^2} + \frac{1}{\sigma_y^2} - \frac{1}{\sigma_b^2} \right) q = \frac{2r_b(\sigma_b - \sigma_y)}{(1 + r_b)\sigma_y^2} - \frac{2r_b\sigma_b}{(1 + r_b)\sigma_y^2} + c$$

(Eq 11)

In the above equations, $\sigma_x$ and $\sigma_y$ are yield stresses for uniaxial tension at 0° and 90° to the rolling direction, respectively, and $r_b$ and $r_y$ are ratios of transverse to through-thickness strain corresponding to $\sigma_b$ and $\sigma_y$, respectively.

Similar to the definition of $\alpha_b$, it is assumed that the ratio of the yield stresses $\sigma_x$ and $\sigma_y$ also remains constant so that $\sigma_x = \alpha_x\sigma_y$. Then, Eqs 6 to 8 can be written as

$$\frac{\sigma_x^2}{\sigma_y^2} - \alpha_x \frac{c\sigma_y^2}{\sigma_y^2} + \alpha_x \frac{\sigma_y^2}{\sigma_y^2} + \left\{ (p + q) - \alpha_x \frac{p\sigma_1 + q\sigma_2}{\sigma_y^2} \right\} \frac{\sigma_1\sigma_2}{\sigma_x\sigma_y} = 1$$

(Eq 12)

where

$$c = \frac{1}{\alpha_x} + \alpha_x - \frac{\alpha_x^2}{\sigma_0}$$

(Eq 13)