An Advanced Model of Designing Controlled Strain Rate Dies for Axisymmetric Extrusion

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This paper presents an advanced model for the design of stream-lined axisymmetric extrusion dies based on a prescribed strain rate variation. This is vital to the preparation of the workpieces with mechanical properties that are very sensitive to the strain rate distribution during a manufacturing process. The proposed model, which incorporates Tresca’s yield criterion and velocity field with the die angularity, can give an accurate prediction of the die shape. Influences of the interfacial friction and the ram velocity on the die geometry are also studied. As a verification of the proposed model, an updated Lagrangian formulated, elasto-plastic finite element program was developed to analyze the axisymmetric extrusion process. A clear derivation of the load-correction matrix, which is indispensable for the surface traction rate equilibrium in the updated Lagrangian formulation, is described for the application of the finite element simulation. A friction-correction matrix based on a constant shear law is used to solve the interfacial friction. From the comparison of the resultant strain rate distribution, it verifies that the advanced model can determine the surface angularity and friction force in the extrusion process.

Keywords advanced model, axisymmetric extrusion, constant strain rate

1. Introduction

Most materials can be manufactured via extrusion process for producing shaped products. However, extrusion technology has undergone limited improvement in recent years due to the inherent complexity of the process. Especially for some composite materials, the strain rate variation experienced in the material as it flows through the die is an important factor in the extrusion process. The strain rate must be controlled under a certain magnitude so that the influences of the generation and transfer of heat, as well as the distortion of the material on the metallurgical variations, are acceptable.

Conventional extrusion dies, such as the flat face (shear) dies and converging dies, have been used in the extrusion for several decades. Because of the hot shortness resulting from considerable adiabatic shear heating, the shear dies can only be operated at low ram speeds. Conversely, converging dies, which have conical, parabolic, or streamlined shapes, are better suited to generate smoother material flow. According to the study carried out by Srinivasan et al. (Ref 1), when conical dies are applied, the flowing material is subjected to rigid body rotations and abrupt changes in velocity at the entrance and exit planes of the die. This yields great redundant work. The situation of a parabolic die is similar. As for streamlined dies, such as cubic spline dies, the material flow path is smooth and the velocity is continuous, but the strain rate generally peaks toward the exit (Ref 1). Obviously, a new technique of developing adequate shapes for controlling strain rate is necessary in the extrusion process.

Srinivasan et al. (Ref 1) developed an ideal work slab method to predict a die profile, which can have a controlled strain rate in extrusion. However, their methodology ignores the influences of surface angularity and interfacial friction force on the material flow. Hence it cannot apply to the processes with high friction stress and sloped tool-workpiece contacted surface. Chang (Ref 2) recently proposed a new advanced slab method, which combines the velocity fields with the yield criterion, for plane strain strip rolling process. In Chang’s method, Mohr’s circle is used so the stress state can involve the effects of the surface angularity and friction force. In this paper, Chang’s method will be modified and applied to the design of strain rate driven dies for the axisymmetric extrusion process.

As a verification for the developed advanced slab method, an updated Lagrangian formulated, elasto-plastic finite element program was also constructed to analyze the axisymmetric extrusion process. Due to the change of the configuration of the tool-workpiece surface, the equilibrium of the surface traction rate induced the load-correction matrix (Ref 3, 4). The theoretical formulation of the load-correction matrix for axisymmetric process was derived in this work. Thereafter, the friction-correction matrix of constant friction shear law was derived to solve the frictional traction force rate. After incorporating these two correction matrices into the usual updated Lagrangian formulated, finite element analysis, the axisymmetric extrusion was simulated completely from the initial nonsteady state to the steady state. The resultant strain rate distribution of the advanced model varies so that it can be applied to solve the surface angularity and friction force.

2. Theory

2.1 Advanced Slab Model

As shown in Fig. 1, the material is pushing through a convergent axisymmetric die with the inlet diameter \( h_1 \) and the out-
let diameter $h_2$. The length of the die is $L$. If the material undergoes a prescribed strain rate, it is convenient to use the Mohr’s circle technique to describe the shear stress distribution including angularity effect. Knowing that the state of stresses at any point under plastic deformation satisfies the Tresca yield criterion, the radius of this Mohr’s circle equals the shear yield strength of the workpiece, say $k$. It is written as

$$(\sigma_x - \sigma_r)^2 + 4\tau_{xr}^2 = 4k^2 \quad \text{(Eq 1)}$$

In the constant friction law, the shear stress along the tool-workpiece interface can be expressed as

$$\tau = -mk \quad \text{(Eq 2)}$$

where $m$ is the friction factor. If the friction angle, $\alpha$, is defined as

$$\alpha = \sin^{-1}(-mu) \quad \text{(Eq 3)}$$

then the shear stress on the tool-workpiece interface with respect to the $x$-$r$ coordinate system is obtained geometrically from Mohr’s circle

$$\tau_{xr}(x,h/2) = k \sin(\alpha + 2\lambda) \quad \text{(Eq 4)}$$

where $\lambda$ represents the geometrical angularity of process. That is

$$\lambda = \tan^{-1}\left(\frac{1}{2} \frac{dh}{dx}\right) \quad \text{(Eq 5)}$$

Because the shear stress $\tau_{xr}$ vanishes at the centerline of the workpiece, it is assumed that the shear stress varies linearly along the thickness of the workpiece and can be expressed as

$$\tau_{xr}(x,r) = \frac{2kr \sin \phi}{h} \quad \text{(Eq 6)}$$

where

$$\phi = \alpha + 2\lambda \quad \text{(Eq 7)}$$

After substituting Eq 6 into Eq 1, the following equation is obtained:

$$\sigma_x - \sigma_r = 2k \sqrt{1 - \left(\frac{2r}{h}\sin \phi \right)^2} \quad \text{(Eq 8)}$$

According to the Levy-Mises flow, the relation of the strains rate to stresses is described as

$$\dot{\varepsilon}_x - \dot{\varepsilon}_r = \frac{\sigma_x - \sigma_r}{\tau_{xr}} \quad \text{(Eq 9)}$$

Using the definition of strains rate to the velocity components, an equation that combines the velocity components to the stresses is derived as

$$\frac{\partial u}{\partial r} + \frac{\partial v}{\partial x} = \frac{6}{2} \frac{\partial v}{\partial r} \tau_{xr} \frac{\sigma_x - \sigma_r}{\tau_{xr}} \quad \text{(Eq 10)}$$

where $u$ and $v$ are the velocity components in the global $x$ and $r$ coordinates. Moreover, it is assumed that the vertical component $v$ varies linearly across the workpiece:

$$v(x,r) = \frac{u_r}{h} \frac{dh}{dx} \quad \text{(Eq 11)}$$

where $u_r$ is the horizontal component of velocity on the tool-workpiece interface. Combining Eq 6, 8, and 11, the general expression of $u$ can be derived to be

$$u(x,r) = \frac{1}{16} \left[ 1 - 8 \left(\frac{r}{h}\right)^2 \right] [u_r h' + u_x (hh'' - h'^2)]$$

$$+ \frac{3u_r h'}{2 \sin \phi} \sqrt{1 - \left(\frac{2r}{h} \sin \phi \right)^2} - C \quad \text{+ \frac{4q}{\pi h^2} \quad \text{(Eq 12)}}$$

where $'$ means the derivative with respect to $x$ direction. $q$ is the volume flow rate which is:

$$q = \pi \left(\frac{h_1}{2}\right)^2 u_r = \pi \left(\frac{h_1}{2}\right)^2 (u_1)_{ave} \quad \text{(Eq 13)}$$

where $u_r$ is the ram velocity, which equals the average entrance velocity $(u_1)_{ave}$. $C$ is denoted as the inhomogeneity function and is derived as