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Time-dependent response of laminated isotropic strips
with viscoelastic interfaces*

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Abstract: The two dimensional problem of simply supported laminated isotropic strips with viscoelastic interfaces and under static loading was studied. Exact solution was derived based on the exact elasticity equation and the Kelvin-Voigt viscoelastic interfacial model. Numerical computations were performed for a strip consisting of three layers of equal thickness. Results indicated that the response of the laminate was very sensitive to the presence of viscoelastic interfaces.

Keywords: Viscoelastic interfaces, Isotropic laminated strips, Exact solution

INTRODUCTION

A traditional premise in the theory of composites is the continuity of tractions and displacements at constituent interfaces. However, the interfaces are generally weaker than the plies in laminates, which then frequently suffer from failure (such as delamination, interlaminar slip, etc.) due to high stress concentration at the interfaces. Before the final failure, these interfaces are usually weakened and become imperfect due to the emergence of microcracks. There are numerous papers in the literature dealing with various aspects of composites with imperfect interfaces (Benveniste, 1985; Hashin, 1991a; Zhong and Meguid, 1996; Yu and Zhong, 1999; Shu and Soldatos, 2001). In most works, such as those mentioned above, the response of laminates under static loading does not vary with time. More recently, He and Jiang (2003) derived an exact two-dimensional solution for isotropic laminates with viscous interfaces and showed that the response of laminates varies remarkably with time, especially at the initial stage. Chen and Lee (2004) proposed an efficient and accurate semi-analytical method for the analysis of angle-ply laminates in cylindrical bending with viscous interfaces. Both studies revealed that, as time approaches infinity, the viscous interfaces will lose the ability of transferring shear stress totally. However, this seems unsuitable for certain types of practical composites, especially within the framework of small deformation. According to Hashin (1991b), a viscoelastic interface will be more appropriate for characterizing the creep and relaxation behavior of interlaminar bonding material under high temperature circumstance.

In this paper, we discuss the two-dimensional responses of a simply-supported laminated isotropic strip (or rectangular plate in cylindrical bending) with viscoelastic interfaces of Kelvin-Voigt type model, subjected to sinusoidal loading. As a primary exploration, we assume that each layer in the strip is elastically isotropic. An
exact solution is derived and numerical results are given and discussed.

SOLUTION PROCEDURE

Consider an \( n \)-layered simply-supported laminated isotropic strip (plate in cylindrical bending) as shown in Fig.1. The strip has width of \( l \), and is simply supported at \( x=0 \) and \( x=l \). The Young’s modulus and Poisson’s ratio of the \( k \)th layer are \( E_k \) and \( \mu_k \) respectively. The constitutive law of the \( k \)th layer is then written as (He and Jiang, 2003)

\[
\begin{align*}
\sigma_x^{(k)} &= \frac{E_k}{(1-2\mu_k)(1+\mu_k)} \left[ (1-\mu_k) \frac{\partial u^{(k)}}{\partial x} + \mu_k \frac{\partial w^{(k)}}{\partial z} \right] \\
\sigma_z^{(k)} &= \frac{E_k}{(1-2\mu_k)(1+\mu_k)} \left[ (1-\mu_k) \frac{\partial w^{(k)}}{\partial z} + \mu_k \frac{\partial u^{(k)}}{\partial x} \right] \\
\tau_{xz}^{(k)} &= \frac{E_k}{2(1+\mu_k)} \left( \frac{\partial u^{(k)}}{\partial z} + \frac{\partial w^{(k)}}{\partial x} \right)
\end{align*}
\]

where \( u \) and \( w \) are the displacements in \( x \)- and \( z \)-directions, respectively. \( \sigma_x \) and \( \sigma_z \) are the normal stresses, and \( \tau_{xz} \) is the shear stress. The equilibrium equations can be written in terms of displacements as

\[
2(1-\mu_k) \frac{\partial^2 u^{(k)}}{\partial x^2} + (1-2\mu_k) \frac{\partial^2 u^{(k)}}{\partial x \partial z} + \frac{\partial^2 w^{(k)}}{\partial z^2} = 0
\]

\[
2(1-\mu_k) \frac{\partial^2 w^{(k)}}{\partial z^2} + (1-2\mu_k) \frac{\partial^2 w^{(k)}}{\partial x^2} + \frac{\partial^2 u^{(k)}}{\partial x \partial z} = 0
\]

Considering a sinusoidal loading \( p=p_0 \sin(\alpha x) \) \((\alpha=\pi/l)\) applied on the top surface, we will solve Eq.(2) under the following boundary and interfacial conditions

\[
\begin{align*}
\sigma_z^{(e)} &= -p_0 \sin \alpha x, \quad \tau_{xz}^{(e)} = 0, \text{ at } z = h \\
\sigma_z^{(i)} &= 0, \quad \tau_{xz}^{(i)} = 0, \text{ at } z = 0 \\
\sigma_z^{(k)} &= w^{(k)} = 0, \text{ at } x = 0 \text{ and } x = l \\
\sigma_z^{(k+1)} &= \sigma_z^{(k)}, \quad \tau_{xz}^{(k+1)} = \tau_{xz}^{(k)}, \\
u^{(k+1)} = u^{(k)} + \delta^{(k)}, \quad w^{(k+1)} = w^{(k)}, \text{ at } z = z_k
\end{align*}
\]

where \( \delta^{(k)} \) is the relative sliding displacement at the \( k \)th interface. In this paper, we assume that the shear stress and sliding obey the Kelvin-Voigt viscoelastic law

\[
\tau_{xz}^{(k)} = \eta_0^{(k)} \dot{\delta}^{(k)} + \eta_1^{(k)} \ddot{\delta}^{(k)},
\]

where \( \dot{\delta}^{(k)} \) is the sliding velocity (the dot over a quantity denotes differentiation with respect to time), and \( \eta_0^{(k)} \) and \( \eta_1^{(k)} \) are the elastic constant and viscous coefficient, respectively. Setting \( \eta_0^{(k)} = 0 \), we get the viscous model studied by He and Jiang (2003). The solution to Eq.(2) had already been derived by He and Jiang (2003) as

\[
\begin{align*}
u^{(k)} &= f^{(k)} \cos \alpha x, \quad w^{(k)} = g^{(k)} \sin \alpha x, \\
\end{align*}
\]

with

\[
\begin{align*}
f^{(k)}(z) &= (C_1^{(k)} + zC_2^{(k)})e^{az} + (C_3^{(k)} + zC_4^{(k)})e^{-az}, \\
g^{(k)}(z) &= \frac{1}{\alpha} \left[ (\alpha C_1^{(k)} + (\alpha z - 3 + 4\mu_k)C_2^{(k)})e^{az} - \frac{1}{\alpha} (\alpha C_3^{(k)} + (\alpha z + 3 - 4\mu_k)C_4^{(k)})e^{-az} \right]
\end{align*}
\]

where \( C_i^{(k)} \) are integral constants to be determined. The corresponding expressions for stresses are

\[
\begin{align*}
\sigma_x^{(k)} &= -\frac{E_k}{1+\mu_k} \left[ (\alpha C_1^{(k)} + (\alpha z + 2\mu_k)C_2^{(k)}) e^{az} \\
&+ (\alpha C_3^{(k)} + (\alpha z - 2\mu_k)C_4^{(k)}) e^{-az} \right] \sin \alpha x,
\end{align*}
\]