Numerical analysis of surface plasmons excited on a thin metal grating*

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Abstract: The authors numerically investigated the characteristics of surface plasmons excited on a thin metal grating placed in planer or conical mounting. After formulating the problem, the solution method, Yasuura’s method (a modal expansion approach with least-squares boundary matching) was described. Although the grating is periodic in one direction, coupling between TE and TM waves occurs because arbitrary incidence is assumed. This requires the employment of both TE and TM vector modal functions in the analysis. Numerical computations showed: (1) the excitation of surface plasmons with total or partial absorption of incident light; (2) the resonance character of the coefficient of an evanescent order that couples the plasmon surface wave; (3) the field profile and Poynting’s vector. The plasmons excited on the surfaces of a thin metal grating are classified into three types: SISP, SRSP, and LRSP, different from each other in the feature of field profile and energy flow. In addition, the eigenvalue of a plasmon mode was obtained by solving a sequence of diffraction problems with complex-valued angles of incidence and using the quasi-Newton algorithm to predict the real angle of incidence at which the absorption occurs.

Key words: Thin metal grating, Plasmon modes, Resonance absorption, Numerical analysis


INTRODUCTION

It is widely known that a metal grating supports plasmon surface waves in the optics region (Raeter, 1982; Nevier, 1980). Excitation of a plasmon surface wave causes resonance absorption, which can be observed as partial or total absorption of incident light accompanied by an abrupt change of diffraction efficiency called resonance anomaly.

If the grating is thick enough, the surface waves can be excited on the lit surface alone and the surface waves in this case are called single-interface surface plasmon (SISP). While if the grating is thin, which is the case we are interested in, simultaneous excitation of plasmon surface waves on both the surfaces occurs.

The plasmons interact with each other to form two types of coupled plasmon modes: a short-range surface plasmon (SRSP) and a long-range surface plasmon (LRSP).

When a thin metal grating is illuminated by a monochromatic plane wave, resonance absorption occurs at the angles of incidence at which the SRSP and LRSP are excited. The angles are called resonance angles and are determined by the phase-matching condition: The phase constant of an evanescent order agrees with the real part of an eigenvalue of a plasmon mode. Because the coupled modes are different in eigenvalues, two absorption dips corresponding to two coupled modes are observed.

In the following sections we first formulate the problem of diffraction by a thin metal grating assuming that the grating is placed in conical mounting. That is, the plane of incidence spanned by the incident wavevector and the grating normal is assumed to
make an arbitrary azimuth angle with the direction of periodicity. If the azimuth is zero, the placement is termed planer or classical mounting. In a planer mounting case the resonance absorption can be seen for TM-wave (p polarization) incidence alone. While in a conical mounting case, it is observed for both TE-wave (s polarization) and TM-wave incidence.

After describing the problem, we introduce briefly the solution method: Yasuura’s method (Yasuura and Itakura, 1965; 1966a; 1966b; Yasuura, 1971; Okuno, 1990). It is a modal expansion approach combined with least-squares boundary matching. Approximate solutions inside and outside of the metal layer are defined in terms of linear combinations of vector modal functions with unknown coefficients. The coefficients are determined in order that the solutions satisfy the boundary conditions in the least-squares sense.

On the other hand, by solving a sequence of diffraction problems with complex-valued incident angles and using the quasi-Newton iteration algorithm, we can find the eigenvalues of the plasmon modes. Having the eigenvalues of a grating, we can predict the resonance angles by applying the phase-matching condition.

Numerical computations were carried out for a relatively thick (the thickness to period ratio is 0.4 and the period is a little less than the wavelength) and two thin gratings (the ratio is 0.08 or 0.02). After observing the resonance absorption as dips in efficiency curves, we show the resonant character of the modal coefficient of an evanescent order, the field distribution, and energy flow. Besides, we illustrate the thickness dependence of the eigenvalues obtained by the iteration procedure. Prediction of the resonance angles from the eigenvalues agrees well with the numerical result for the diffraction problems.

FORMULATION OF THE PROBLEM

Fig.1 illustrates the schematic representation of diffraction by a thin metal grating. The grating is uniform in y and is corrugated in x with a period d. The grating has an upper and a lower surface, which are given by

\[ \begin{align*}
S_1: & \quad z_1 = \eta_1(x) = h \sin(2\pi x / d), \\
S_2: & \quad z_2 = \eta_2(x) = \eta_1(x) - e,
\end{align*} \]

where e and h are thickness and amplitude of the grating. Note that (x,z) denotes a point on the surface of the grating while \( P=(X,Z) \) denotes a point inside a region. The surfaces separate the whole space into three regions: \( V_1 \) \( [ Z>\eta_1(X)] \), \( V_2 \) \( [ \eta_1(X)>Z>\eta_2(X)] \), and \( V_3 \) \( [ Z<\eta_2(X)] \). We assume that the region \( V_2 \) is occupied by a metal with a complex-valued refractive index \( n_2 \) and that regions \( V_1 \) and \( V_3 \) are vacuums \((n_1=n_3=1)\).

The electric and magnetic field of an incident light are given by

\[
\begin{bmatrix}
E^i \\
H^i
\end{bmatrix}(P) = \begin{bmatrix}
e^i \\
h^i
\end{bmatrix} \exp(jk^i \cdot P - j\omega t),
\]

and

\[ h^i = (1/\omega \mu_0)k^i \times e^i, \]

here, \( e^i \) is the electric field amplitude, \( h^i \) is the magnetic field amplitude and \( k^i \) is the incident wavevector.