Curvature detail representation of triangular surfaces*

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Abstract: Curvature tells much about details of surfaces and is studied widely by researchers in the computer graphics community. In this paper, we first explain the mean-curvature view of Dirichlet energy of triangular surfaces and introduce a curvature representation of details, and then present surfaces editing applications based on their curvature representation. We apply our method to surfaces with complex boundaries and rich details. Results show the validity and robustness of our method and demonstrate curvature map can be a helpful surfaces detail representation.

Key words: Curvature, Detail representation, Mesh editing, Shape editing, Dirichlet energy

INTRODUCTION

With the favorites of finer and vivid 3D models in visual reality, games and entertainment community, researchers among academic institutions and industrial organizations construct detail-rich models by 3D scanning tools and geometry modelling. These models usually contain millions of triangles and occupy hundreds and thousands of MB memories which give great difficulties for editing or operating onto them. A good representation of details makes these operations much easier. In this paper, we propose a detail representation of surfaces by its mean curvature information. Not just aiming to simplify mesh operations, our curvature detail representation has clear geometric explanation.

Previous work

First, we will list the technologies about detail representation, which are mainly used in mesh editing, shape deformation and animation. Second, curvature computation method and its applications in computer graphics are discussed.

1. Detail representation

In 3D triangular mesh space, geometry details can be represented in many ways. Ju et al. (2005) computed vertices weights by mean value coordinates method in 3D space onto a bounding coarse control mesh. The 3D mean value coordinates method is well-defined on both interior and exterior of the mesh surface and the deformation is real-time. Poisson equation-based method is another useful detail representation scheme and is studied in geometry completion, detail generation, mesh editing and deformation, etc. Poisson equation has its complex physical explanation but can be applied directly, on 2D images (Pérez et al., 2003) or on 3D meshes (Yu et al., 2004; Nguyen et al., 2005). A parameterization of 2D pixel RGB or 3D coordinates onto a planar region is needed to get the gradient map of the surface function \( f \). With this boundary condition and guidance gradient map a target image or surface is constructed. Here gradient
map works as details information in Poisson equation. Laplacian based method is also widely used as detail representation for mesh editing (Nealen et al., 2005; Au et al., 2005; Sorkine et al., 2004) or smoothing (Desbrun et al., 1999). In certain meaning, our mean curvature detail representation can be classified in this catalogue. But we explain the detail not as the displacement to its neighborhood but with a curvature meaning. Laplacian based method usually includes a sparse linear system, which makes it easy to operate and fast to compute. Igarashi et al. (2005) uses triangle shape and scales as the details of 2D triangular meshes which should be preserved during deformation. Lipman et al. (2005) represents the details as difference between local frames for neighbor vertices and deforms the object while keeping its original frame differences. All these detail representation methods are practically efficient and some of them have their geometrical explanations. The detail representation proposed in this paper relates curvature of surfaces and Dirichlet energy of its triangular approximation.

2. Curvature computation and applications

In differential geometry, curvature is mainly studied on continuous surfaces. To deal with triangular meshes, Meyer et al. (2002) defines mean curvature normal operator and Gaussian curvature operator with equations discretized from continuous surfaces. Other schemes base on curvature tensor method (Taubin, 1995) or on the theory of normal cycles (Cohen-Steiner and Morvan, 2003). We use scheme (Meyer et al., 2002) in which curvature can be formulated as a linear system of vertex position, simplifying latter operations. Curvature guides many applications in computer graphics, for mesh segmentation of CAD models (Guillaume et al., 2004) and improves mesh parameterization of general meshes (Yamauchi et al., 2005), for mesh saliency (Lee et al., 2005), for mesh fairing while keeping desirable geometric features (Alliez et al., 2003; Desbrun et al., 1999). But curvature cannot be used in mesh editing easily as it is not as intuitive as vertex position. This paper proposes a mesh editing method guided by its mean curvature representation.

CURVATURE DETAIL REPRESENTATION

Definitions and notations

In continuous 3D surface space, curvature is defined as the derivative of the tangent vector of space lines. Mean curvature is half of the sum of the two principal curvatures \(k_1\) and \(k_2\) of a point \(x_i\) on a surface. Its discretization on triangular meshes is

\[
K_i = \sum_{j \in F_i} (\cot \alpha_{ij} + \cot \beta_{ij}) (x_i - x_j) / 2A_i,
\]

where \(K_i\) is the mean curvature normal operator.

For a surface \(f : \mathbb{R}^2 \rightarrow \mathbb{R}^3\) parameterized on a close region \(\Omega \subset \mathbb{R}^2\), usually an energy function is present as global metric. For most energy functions, Dirichlet energy function is among the most frequently used and is defined as \(E_D = \int_{\Omega} |\nabla f|^2 / 2\), where \(\nabla f\) is the gradient operator of \(f\). Its discretization is

\[
E_D(M) = \left\{ \sum_{(i,j) \in \mathcal{E}} (\cot \alpha_{ij} + \cot \beta_{ij} | x_i - x_j |) \right\} / 4.
\]

\[
\partial E_D(M) / \partial x_i = \left\{ \sum_{j \in F_i} (\cot \alpha_{ij} + \cot \beta_{ij}) (x_i - x_j) \right\} / 2 \text{ is its derivative.}
\]

The discretization of mean curvature and Dirichlet energy are deduced many ways (Meyer et al., 2002; Desbrun et al., 1999; 2002; Pinkall and Polthier, 1993), but they use them for different purposes.

Base presentation

From previous discussion, it is obvious that \(\partial E_D(M) / \partial x_i = -K_iA_i\). This form views curvature as global Dirichlet energy metric for triangular meshes. It shows that the vertex differential of Dirichlet energy is twice the product of its mean curvature normal and area operators, and that it is an intrinsic property for meshes. Based on this analysis, we propose a curvature detail representation for triangular meshes and assume this detail representation contains the local and global information. Not like other proposed methods just aimed at practicality, our method has firm mathematical basic.

For an unknown mesh \(M\) and its guidance mean curvature normal operator \(k\), we aim to construct the mesh surface \(f\) with its vertex derivative of Dirichlet energy equaling to \(K\). This is done quadratically by minimizing function \(E_Q(f) = \int \| \partial E_D(f) / \partial x_i \| \). When \(f\) is discretized triangularly with a certain vertex connection, minimizing \(E_Q\) is equal to minimizing a linear system \(\Phi^T d\), which can be solved.