Model Calculations in Reconstructions of Measured Fields

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Abstract: The state of technological systems, such as reactions in a confined volume, are usually monitored with sensors within as well as outside the volume. To achieve the level of precision required by regulators, these data often need to be supplemented with the solution to a mathematical model of the process. The present work addresses an observed, and until now unexplained, convergence problem in the iterative solution in the application of the finite element method to boundary value problems. We use point group theory to clarify the cause of the non-convergence, and give rules for choosing the appropriate and consistent orders of approximation on the boundary and within the volume so as to avoid non-convergence.

Keywords: boundary value problem, reactor physics, point group theory, convergence problem
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1 Introduction

The state of an industrial process distributed over a volume, such as, for example, a reaction rate and/or flow in a tank, is usually monitored for safe and economic operation through a system of measuring devices placed within and/or on the boundary of the volume. The inference of the state of the system from such measurements is the classical mathematical inverse problem. This problem, in general, has no unique solution and

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requires additional information, in the form of a priori properties of the solution consistent with the specific physical setting. Thus, the accuracy of the solution in a particular instance is dependent on both measurements and the solution of model calculations of the physical setting. If measurements both in abundant quantity and high accuracy are available, the need for added information from a model calculation of the process is not indispensable. In many industrial process, however, the measurement environment is inimical to both measurement instrumentation and accurate results. An example of this is the monitoring of the power distribution, temperature and coolant flow in the core of a nuclear power reactor (Makai, Temesvári and Orechwa, 2001). In such systems, where not only the number of sensors is low due to cost, but also the failure probability is high, great reliance is placed on supplementing the measurements with sophisticated and highly accurate model calculations. These calculations usually involve the numerical solution of a boundary value problem.

We address the solution of the model problem, in the context of a very widely used particular case of the general class of weighted residual methods for the numerical solution of boundary value problems, namely the finite element method (Strang and Fix, 1973). The method is based on subdividing the volume of interest into smaller volumes called nodes. Each node is generally considered to be homogeneous in composition. Central to the accuracy of the method are two approximations. In the first, we assume the solution on the boundary surface of the node to be expanded in a set of basis functions \( (f_i(\xi), i = 1, ..., N) \). In the second, the solution inside the volume is expanded in another set of basis functions \( (F_j(x), j = 1, ..., M) \). Clearly the independent variable \( \xi \) is a limit of the independent variable \( x \).

Any iteration procedure, in principle, connects neighboring nodes through continuity and smoothness conditions. For an efficient numerical algorithm it would therefore be desirable to have the same number of degrees of freedom (i.e. coefficients in the expansion) on the surface of the node as within the node. In Table 1, for the case of a square node, we compare the required number of coefficients for different orders of polynomial expansion. It is clear that there is no order of approximation which exhibits the same number of coefficients on the surface as inside the node. The appropriate choice of order of expansion is thus not straightforward but it is important to the accuracy of the solution, because a mismatch of degrees of freedom inside and on the surface of the node is likely to lead to a loss of information in the computational step that passes from one node to the next. This has been observed, see Palmiotti et al., (1995) and Lewis et al. (1996), in the case of calculations with a square node and first order polynomials on the surface. In that analysis a convergent solution is obtained only for fourth or higher order polynomials on the inside of the node.

Similar relationships apply to other nodes as is shown in Table 2 for the case of a regular hexagon. Again there is no polynomial order where the number of coefficients on the surface matches the number of coefficients on the inside of the node. In addition, finite element solutions based on a hexagonal node Carrico, Lewis and Palmiotti (1994), Palmiotti et al. (1995), have demonstrated that the first convergent solution with a linear