Comparison Study on Algorithms for Vehicle Routing Problem With Time Windows

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Abstract - This paper investigates and compares heuristic algorithms for the vehicle routing problem with time windows. Three heuristic algorithms were proposed firstly through combining three classic heuristic algorithms with the cross exchange method (cross) and the 2-opt exchange heuristic (2-opt) respectively. These proposed algorithms are then compared with the three classical algorithms based on publicly available benchmark problems. The comparison results show that the effectiveness of the proposed algorithms and their superiority to the classical algorithms.

Keywords - Algorithm comparison, heuristic algorithms, Solomon benchmark problem, vehicle routing problem with time windows

I. INTRODUCTION

The vehicle routing problem with time windows (VRPTW) is developed based on the basic vehicle routing problem (VRP) through adding a time window constraints. Recently, with the development of e-commerce distribution, the academic research and practical application of vehicle routing problem with time windows got much attention from industry and academia.

A large number of papers have been published in the VRPTW area. Some researchers have provided comprehensive surveys [1-4]. Early research of VRPTW mainly includes exact algorithms and classical heuristic algorithms. At present, global optimal solutions to VRPTW can be barely found by exact algorithms except for small-scale problems [5-6]. When the problem size is bigger, heuristic algorithms are usually used to find "nearly optimal solutions" or "satisfied solutions" since exact algorithms cannot find the optimal solutions within a reasonable time period. In the literature, three classical heuristics are commonly used, which are the savings algorithm [7], the sweep algorithm [8] and the insertion algorithm [9].

The core idea of savings algorithm is to combine two loop of the transportation problem into one circuit in turn, according to the principle of maximize the reduce of total distance. Optimize a new car until the current car arrived the load limit. Optimization process of savings algorithm can divide into parallel and serial manner.

Sweep algorithm was proposed by Gillett and Miller in 1974. Sweep algorithm has two phases of steps, which adopts polar coordinates to indicate the location of demand point, and then take a demand point as the starting point randomly, and make its Angle as zero. Then divide the service area according to the constraints of car capacity in clockwise or the reverse direction of the clock, and rank the demand points by Lin and Kemighan exchange method, thus to construct the vehicle route schedule.

Insertion method is also called "farthest insertion method", which was proposed by Mole and Jameson in 1976 for solving vehicle routing problem. This method combined the idea of approaching method and savings method, insert customers into path one by one to build distribution route. This method firstly put the farthest point of terminal as the seed point of route, then take the minimum insert value point as the next point according to the concept of the adjacent point insertion method. Finally it decides the insert location according to the saving value generated from the general saving formula. Repeat the steps of selection and insert until vehicle capacity or time window is meet limit.

Some researchers also developed a large number of intelligent heuristic algorithms to solve the VRPTW, such as tabu search, genetic algorithm, simulated annealing algorithm and ant colony algorithm. Braysy and Gendreau [10-11] reviewed classic heuristic algorithms and intelligent heuristic algorithms for the VRPTW.

The classic heuristic algorithm seek satisfied solution by local search technology, it is simple and easy to realize, but easy to fall into local optimum. The intelligent heuristic algorithms are inspired by the idea of artificial intelligence, some of which have the potentials of finding globally optimal solutions through global search methods. However, the parameter settings of these algorithms are very complex and usually problem-dependent. Unfortunately, the parameter settings have a greater influence on the performance of algorithm. It is thus hard for these algorithms to be used in practice.

Comparing with intelligent heuristic algorithms, classical heuristic algorithms are usually easy-to-understand and easy-to-implement. However, the comparison study on classical heuristic algorithms has not been conducted before.

This paper thus presents a comparison study on heuristic algorithms for the VRPTW. Three novel heuristic algorithms are proposed firstly by combining three classical heuristic algorithms with the cross and the 2-opt respectively. The proposed algorithms are then evaluated and compared with the three classical algorithms based on publicly available benchmark problems.

The remainder of this paper is organized as follows. In section II, a simple introduction and mathematical model of the vehicle routing problem with time windows (VRPTW) is given. Three new heuristic algorithms are presented in section III. In section IV, we analyzed and
compared the experiment results of the proposed algorithms and three classical algorithms. Concluding remarks, along with further research possibilities are given in the final section.

II. VEHICLE ROUTING PROBLEM WITH TIME WINDOWS

The VRPTW is an extension of the classical VRP. In the VRPTW, nodes are associated with more properties, and the solution has to satisfy more constraints. A service time $s_i$ is considered; therefore, the vehicle has to stay at the location of customer $c_i$ for a time interval at least $s_i$ ($s_0 = 0$ is associated with the depot $c_0$) for service. A time window $[e_i, l_i]$ during which the service has to start is considered. Therefore, when a vehicle arrives at customer $c_i$ earlier than $e_i$, it has to wait until the beginning of the time window to serve the customer. On the other hand, if a vehicle cannot arrive at $c_i$ before $l_i$, the vehicle cannot serve $c_i$. At this time, customer $c_i$ should be served by another vehicle. For depot $c_0$, the time window is defined as that $e_0$ is the earliest start time, and $l_0$ is the latest return time of all the vehicles. The VRPTW has two objectives. The primary objective is to minimize the number of the vehicle routes $V$. The secondary objective is to minimize the total TD with the same number of routes. Constraint (3)-(4) denotes that every route only one vehicle pass. Constraint (5) denotes that the quantity of goods that each vehicle carries could not exceed the capacity. Constraint (6) stands for that each customer can only be served by one vehicle. Constraint (7) represents that all the routes start from the depot. Formulas (8)-(10) define the time window constraint, where $l_i$ is the time when the vehicle arrives at node $i$; $w_i$ is the waiting time of a vehicle at the customer location until the time $e_i$; $s_i$ is the service time; and $t_{ij}$ is the travel time between nodes $i$ and $j$.

III. NEW HEURISTIC ALGORITHMS

This section describes how to develop the three new heuristic algorithms for the VRPTW by combining savings, sweep or insertion algorithms with 2-opt and cross method respectively.

Fig. 1 shows the flowchart of the improved savings algorithm. The procedures involved are described below:

Step 1. Initialization: Generate initial solutions through savings algorithm;
Step 2. Crossover: Choose an initial path from the initial solution, and operate cross with another initial path;
Step 3. 2-opt Operation: Operate 2-opt to two new paths generated cross crossover operation respectively, find two new path with shortest total length;
Step 4. Judgment of total length: While the total length of the two new paths is shorter than the total length of step 2, replace the initial path with new path; while the total length of the two new paths isn't shorter than the total length of initial solution, retain the initial path;
Step 5. Judgment of crossover: Check whether the cross crossover operation has come to an end. If it's not true, return to step 3; if it's true, carry out step 6;
Step 6. Judgment of initialization: Check whether the entire initial path had carried out cross crossover operation between every two paths. If it's not true, return step 2; if it's true, output the final solution.

Comparing with the improved savings algorithm, the improved sweep algorithm and the improved insertion algorithm have the same procedures except for Step 1. The two improved algorithms generate initial solutions by using sweep and insertion algorithms respectively in Step 1.

\[
\sum_{i=1}^{v} y_0^i = v \tag{7}
\]

\[
t_i + w_i + s_i + t_{ij} = t_j \quad \forall i, j = 0, 1, \ldots, n \neq j \tag{8}
\]

\[
e_i \leq t_i \leq l_i \quad \forall j = 0, 1, \ldots, n \tag{9}
\]

\[
w_i = \max\{e_i - t_i, 0\} \quad \forall j = 0, 1, \ldots, n \tag{10}
\]