Chapter 6

Overview of Each Nations Economic State Dynamics and Control: Equations of GDP, Interest Rate, Employment, Value of Capital Stock, Prices, Cumulative Balance of Payment

6.1 Member Nations’ Economic State and Strategies. Existing Results

The economic state is \( x = [y, R, L, k, p, E] \). We now recall our earlier equations for interest rate [2, p. 29] – Stability and Time Optimal Control of Hereditary Systems with Applications to Economic Dynamics of the USA. Let \( M \) denote money supply (e.g., M1)

\[
\mathcal{L} = \text{money demand} \approx Md
\]

Quasimoney, set

\[
ML = Md - M
\]

Assume

\[
\mathcal{L} = M_0 + M_1y(t-h) + M_3R(t) + M_4R(t-h) + M_5R(t-h) + M_6p(t).
\]

The rate of interest is determined by the typically Kenesian dynamics

\[
\frac{dR}{dt} = \lambda_2(\mathcal{L} - M),
\]

which yields

\[
\dot{R}(t) - a_{-22} \dot{R}(t-h) = a_{21}y(t) + a_{22}y(t-h) + a_{33} \dot{R}(t) + a_{24}R(t-h) + a_{25}p(t) - \sigma_2(t) + q_2(t)
\]

(6.1)

where

\[
q_2(t) = -\lambda_2 M_1, \quad \sigma_2(t) = \lambda_2 M_0
\]

\[
a_{-22} = \lambda_2 M_5, \quad a_{21} = \lambda_2 M_1
\]
Employment dynamics is derived as

\[
\dot{L}(t) - \ell_{-01}\dot{L}(t - h) - \ell_{-03}\dot{y}(t - h) = \ell_0 L(t) - \ell_1 L(t - h) + \ell_2 y(t - h)
+ \ell_4 R(t - h) + \ell_5 L(t - h) + \sigma_3(t) + q_3(t),
\]

where

\[\ell_{-01} = a_{-1} - m(w)a_6, \quad a_3 m(w) = \ell_{-03}\]
\[\ell_0 = a_0\]
\[\ell_1 = -a_1\]
\[\ell_2 = m(w)a_2\]
\[\ell_4 = m(w)a_4\]
\[\ell_5 = m(w)\]
\[\ell_8 = m(w)a_8\]
\[-\sigma_3(t) = m(w) \sigma_4(t)\]
\[q_3(t) = m(w) q_4(t)\]
\[\sigma_4(t) = x_0 + y_10 + I_0\]
\[q_4(t) = g_0 + z_{s13} M - z_{s14} T(t) + z_{s15} e(t) + z_{s16} \tau(t) + z_{s17} d(t).\]


\[
\dot{p}(t) = p(t)\left[(M_6 + p_4)p(t) - p_6 M_1 y(t) - p_6 y(t - h) - p_6 M_1 R(t)\right]
M_4 p_6 R(t - h) - p_6 M_2 R(t - h) + [q_5(t) + \sigma_5(t)] p(t)
\]

where

\[q_5(t) = p_1 p^f(t) \cdot e(t) + p_5 M_1(t) + p_6 M_1(t),\]
\[\sigma_5(t) = p_0 - p_3 n(t) + p_2 w(t) - p_6 M_0(t)\]

and \(M_1, M_3, M_4, M_6\), are identified in (3.1.4) and \(p_1, p_2, p_3, p_4, p_5, p_6\) are identified in (6.1.30) of “Optimal Control of the Growth of Wealth of Nations”, [2]. The dynamics of capital stock is given in [1, (1.10.58)] and [2, (6.1.57)]

\[
\frac{dk(t)}{dt} + a_{-31} k(t - h) - a_{-33} y(t - h) - a_{-36} \dot{L}(t - h) = 30k(t) - a_{31} k(t - h)
\]