Refinement of a Global Model for the Earth’s Gravitational Field using Airborne Gravimetry Data

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Abstract—An algorithm for combining airborne gravimetry data with the data supplied by a global model of the Earth’s gravitational field is considered. The global model is specified by a spherical wavelet decomposition. An optimal guaranteed estimation of the wavelet coefficients for the gravitational field is used.

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INTRODUCTION

The high accuracy and resolution of airborne surveys allow one to separate the short-wave components of gravity anomalies [1]. Because of this fact, airborne gravimetry data are often used to locally refine various approximate global models of the Earth’s gravitational field. At present, there are several space missions (such as GRACE and GOCE) devoted to the development of new high-accuracy second-generation models of the Earth’s gravitational field. New achievements are also observed in the representation of global data and in solving the problem of combining heterogeneous gravimetric data.

Series expansions in spherical harmonic functions are most widely used to represent the data of global models proposed to study the gravitational field. A disadvantage of such a representation consists in the fact that the number of the expansions’ coefficients increases quadratically when the degree of such an expansion increases; for example, the number of the coefficients used in the EGM08 model exceeds four million. In addition, each coefficient depends on the measurements made on the entire Earth’s surface. Another way of specifying the gravitational potential functionals is the use of spherical wavelet decompositions. The corresponding wavelet representations of some harmonic models are now obtained (see EGM96 [2]). The application of spherical wavelet decompositions allows one to store much less number of the coefficients required to compute the characteristics of the gravitational field at a given point.

The collocation method is widely used to locally refine the models of the Earth’s gravitational field [3]. However, this method has the following disadvantages: it is of high computational complexity, whereas the computed estimate depends on a stochastic model of the gravitational field and often turns out to be inadequate.

In this paper we propose an algorithm for combining airborne gravimetry data with the global data given by harmonic spherical wavelet decompositions [2]. We take into account a complex correlation of errors in airborne measurements and do not use any stochastic model of the gravitational field. One of the peculiarities of our approach consists in the fact that our algorithm uses the wavelet coefficients rather than measurements, which significantly reduces the dimensionality of the original problem. For simplicity we consider the spherical approximation of the Earth.

FORMULATION OF THE PROBLEM AND A METHOD OF ITS SOLUTION

Let $E$ be an airborne survey area. It is assumed that $E \subset \Omega_r$, where $\Omega_r \subset \mathbb{R}^3$ is a sphere of radius $r = R + h$, $h =$ const, and $h > 0$. The airborne measurements are made along straight parallel segments of the corresponding trajectory. On such a segment, the airborne gravimetry data are of the form [1]

$$g'_{air}(x) = F_{air}g(x) + \delta g_{air}(x),$$

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where $g$ is the vertical component of the specific gravity force, $F_{\text{air}}$ is a smoothing filter on this segment, $\delta_{\text{air}}$ is the measurement error, and $x = (x_1, x_2, x_3)^T$. It is assumed that the measurement errors are considered as a stationary Gaussian random process; its mathematical expectation is equal to zero and its correlation function is known. It is also assumed that the errors on different segments are independent. The data supplied by the global model are represented as the following spherical wavelet decomposition of the Earth’s gravity force at an altitude $h$ [2]:

$$
g_{\text{glob}}(x) = \sum_{i=1}^{N_2} \omega_{i2} a_{i2}^{\text{glob}} \Phi_2(x, y_{i2}) + \sum_{j=2}^{J_{\text{glob}}} \sum_{i=1}^{N_j} \omega_{ij} c_{ij}^{\text{glob}} \tilde{\Psi}_j(x, y_{ij}).$$  

(1)

Here $a_{i2}^{\text{glob}}$ are the coefficients of the scaling decomposition of the gravity force; $c_{ij}^{\text{glob}}$ are the coefficients of the wavelet decomposition of the gravity force. $\omega_{ij}$ and $y_{ij}$ are the weights and nodes of the quadrature formula in use; $N_2$ and $N_j$ are the dimensions of the grids; $j$ is the scale level; $\Phi_2$ is the scaling function; $\tilde{\Psi}_j$ is the dual wavelet; and $J_{\text{glob}}$ is the maximum level of the wavelet decomposition specified by the minimum length of the measurement wave.

By $J_{\text{air}}$ we denote the maximum wavelet level corresponding to the minimum length of the measurement wave. By $j_0$ we denote the minimum wavelet level of airborne gravimetry data corresponding to the maximum length of the measurement wave. Here we assume that the following inequalities are valid for these levels:

$$J_{\text{air}} > J_{\text{glob}} \geq j_0.$$

The first inequality is valid, since the spatial resolution of the airborne gravimetry data is better. The second inequality is valid, since the frequency spectrum the airborne gravimetry data intersects with the frequency spectrum of the global data. The refinement of the global data according to the airborne gravimetry data is considered as an estimation problem for the wavelet coefficients on the general levels $j_0, \ldots, J_{\text{glob}}$. First we compute the wavelet coefficients $c_{ij}^{\text{air}}$ for the airborne gravimetry data. To accomplish this, we consider the residual gravity force by subtracting the long-wavelength component $g_{\text{low}}^{\prime}$, i.e., the first $j_0 - 1$ levels of the global model (1), from the airborne gravimetry measurements and by equating the residual field outside the survey area $E$ to zero in order to take into account the effect of the “far-field zones” [3]. The following quadrature formula is used [4]:

$$c_{ij}^{\text{air}} = \int_{D_j \cap E} \Psi_j(x, y_{ij}) g_{\text{res}}^{\prime}(x) \, d\omega(x) \approx \sum_{k=1}^{M_j} \omega_{kj} \Psi_j(x_k, y_{ij}) g_{\text{res}}^{\prime}(x_k).$$

Here $g_{\text{res}}^{\prime} = g_{\text{air}}^{\prime} - g_{\text{low}}^{\prime}$ is the residual gravity force, $\Psi_j$ is the wavelet whose support is $D_j$, $\omega_{ij}$ are the weights of the quadrature formula [4], $x_k$ are the grid nodes where the airborne measurements are made, and $M_j$ is the number of nodes in $D_j \cap E$. It is assumed that the function $g_{\text{res}}^{\prime}$ belongs to the Sobolev space [4]. The errors of the computed wavelet coefficients are determined by the formula

$$c_{ij}^{\text{air}} = c_{ij} + \delta c_{ij}^{\text{air}} - \delta c_{ij}^{\text{low}} + \Delta c_{ij}^{E} + \Delta c_{ij}^{\Psi} + \Delta c_{ij}^{\text{discr}}, \quad i = 1, \ldots, N_j, \quad j = j_0, \ldots, J_{\text{glob}},$$  

(2)

where $c_{ij}$ are the true coefficients of the wavelet decomposition for the residual gravity force, $\delta c_{ij}^{\text{air}}$ are the errors caused by the airborne measurements, $\Delta c_{ij}^{E}$ are the errors caused by defining the residual gravity force outside the survey area, $\Delta c_{ij}^{\Psi}$ are the errors caused by the truncation of the wavelets, and $\Delta c_{ij}^{\text{discr}}$ are the discretization errors. Since the wavelets under consideration are harmonic functions outside the above sphere, their support is not compact. The errors $\delta c_{ij}^{\text{low}}$ are related to the errors of the global model of the Earth’s gravitational field by the formula

$$\delta c_{ij}^{\text{low}} = \int_{\Omega_R} \Psi_j(x, y_{ij}) \delta g_{\text{low}}(x) \, d\omega(x),$$

where

$$\delta g_{\text{low}}(x) = \sum_{i=1}^{N_2} \omega_{i2} \delta a_{i2}^{\text{glob}} \Phi_2(x, y_{i2}) + \sum_{j=2}^{J_{\text{glob}}} \sum_{i=1}^{N_j} \omega_{ij} \delta c_{ij}^{\text{glob}} \tilde{\Psi}_j(x, y_{ij}).$$