INTRODUCTION

The objective of this work is the analytical and numerical investigation of Ohm’s law in the current sheet (CS) of the tail of the Earth’s magnetosphere [1], where a large-scale electric field exists and transverse electric current flows. During substorms, the CS contracts to an extremely small thickness on the order of the ion Larmor radius [2]. Since this extremely thin current sheet (TCS) can be the key factor determining the character of geomagnetic perturbations, it seems important to investigate its properties. The first attempts to estimate the scale of the cross-tail electric current and the main parameters influencing the CS structure were made in the 1980s [3]. However, the estimates obtained at that time were rather rough and did not include the contribution of the electron plasma component into the total current. A self-consistent TCS model presented in [4, 5] allows one to obtain self-consistent profiles of the magnetic field, plasma density, and cross–tail current in the Earth’s magnetosphere for a wide range of parameters. However, this model uses a special coordinate system disregarding the electric field. In order to take the large-scale electric field and the electron current into account, the self-consistent profile of the plasma density obtained in [5] was approximated by an analytical formula and the electron currents were analytically represented using the drift approximation [6, 7]. This made it possible to transfer to a coordinate system with a nonzero electric field and obtain analytical and numerical estimates for the total transverse current as a function of the electric field, normal magnetic field, and the ion and electron temperatures. The relative contributions of different plasma components into the total current were also estimated.

1. APPROXIMATION OF THE NUMERICAL RESULTS PROVIDED BY THE SELF-CONSISTENT TCS MODEL

Let us consider the following configuration of electric and magnetic fields characteristic of TCS [5]:

\[ \mathbf{B} = (B_0 \tanh(\frac{z}{L}), 0, B_n), \]  

\[ \mathbf{E} = (0, E_y, E_z(z)). \]  

In addition, \( E_z(z) \to 0 \) for \( Z \to L \), where \( L \) is the characteristic CS width and \( b_n = \frac{B_n}{B_0} \ll 1 \).

It is usually assumed that the \( B_n \) component is maintained by the dipole magnetic field of the Earth and \( B_x \) arises self-consistently from the tail current. The ambipolar electric field \( E_z(z) \) appears due to different dynamics of magnetized electrons and nonadiabatic ions in TCS [5]. The field is determined by a self-consistent potential in the form of a Gaussian decaying at a distance on the order of the layer width \( L \):

\[ \phi(z) = \phi_0 \exp\left(-\frac{\alpha_1 z^2}{L^2}\right), \quad \alpha_1 > 0. \]  

For this approximation of the potential, the electric field appears in the form

\[ \mathbf{E} = \left(0, E_y, E_z \frac{z}{L} \exp\left(-\frac{\alpha_1 z^2}{L^2}\right)\right), \]  

where \( E_z = 2\phi_0 \frac{\alpha_1}{L} \).
Similarly, in line with the numerical self-consistent model, the ion density is approximated by a Gauss function with a maximum at the center of the sheet,

\[ n(z) = n_0 \left(1 + \beta \exp\left(\frac{-\alpha_2 z^2}{L^2}\right)\right), \quad \alpha_2 > 0. \]  

(5)

Here, the parameters \( \phi_0, \beta, \alpha_1, \) and \( \alpha_2 \) were determined using the numerical results of the self-consistent model [5] in such a way that

\[ \delta I = \int (\psi_{\text{num}} - \psi_{\text{approx}}) f(z) dz \leq \gamma, \]

where \( \gamma \) is the accuracy of the drift approximation. The dependences of the parameters \( \beta \) and \( \phi_0 \) on \( \varepsilon = \frac{v_T}{v_D} \) (the ratio of the thermal velocity to the average flow velocity in the moving coordinate system) for \( \frac{T_i}{T_{\|}} = \frac{T_i}{T_{\perp}} = 6 \) and \( b_n = 0.25 \) are shown in Fig. 1.

In the special Hoffmann–Teller coordinate system moving in the positive direction of the X axis with the velocity \( V_{HT} = cE_y/B_n \), the fields have the form

\[ E_z = 0, \quad E_y = 0, \quad E_z'(z) = E_c(z) \]

\[ B_y'(z) = B_s(z), \]

\[ B_z'(z) = \frac{E_z'}{B_n}E_c(z), \quad B_z = B_n. \]

This transformation is valid under the condition \( E_y / B_n \ll 1 \), which actually takes place in the tail of the Earth’s magnetosphere. In this coordinate system, we specify the ion distribution at the CS boundary as the superposition of the distribution functions for ions entering the layer and going outside [5, 8],

\[ f(\mathbf{v}) = f_0^+ \exp\left(-\frac{(v_{\|}^2 + v_D^2)^2}{v_T^2}\right) \]

\[ + f_0^- \exp\left(-\frac{(v_s^2 - v_D^2)^2 + v_T^2}{v_T^2}\right), \]

(6)

where \( T_i = \frac{m_i v_T^2}{2} \) is the ion temperature and \( v_T \) is the thermal velocity of ions in the quiescent coordinate system. The latter differs from the velocity in the moving coordinate system by a small quantity, such that \( v_T^2 = v_T^2 + c^2 E_z^2 / B_L^2 \) (hereinafter, subscript \( L \) denotes quantities at the boundary of the current sheet).

2. CALCULATION OF THE ION CONCENTRATION AND CURRENT DENSITY

Calculation of the ion concentration and current densities is based on the law of momentum flux conservation for the field and particles [6, 7], i.e., on matching the parameters of the ion distribution function with the parameters of the magnetic field.

In the case of stationary fields, the conservation law is written in the form [3]

\[ \frac{\partial}{\partial x_j} (P_{ij} + T_{ij}) = 0, \]

(7)

where

\[ T_{ij} = \frac{1}{4\pi} \left(\frac{E_x^2 + B_z^2}{2} \delta_{ij} - (E_i E_j + B_i B_j)\right) \]

(8)