1. INTRODUCTION

The Problem of Overbooking

Yield Management (Revenue Management) technologies are widely used in various business areas, including the transport industry and tourism [1–3]. The term was introduced by R. Crandall (American Airlines), who called these technologies critical for managing revenues in transport under conditions of the deregulation of prices introduced in the United States. These technologies are designed for the maximization of the yield (profit) via predicting customer behavior, the market segmentation, predicting the demand, optimizing prices for various services, etc.

Yield management technologies are among the basic tools of managing revenue in passenger air transport. American Airlines gained extra revenue of 1.4 bln USD due to using yield management technologies for three years in the late 1980s. Then, yield management technologies started being used in other industries than air transport: hotels, car rentals, cruise lines, mass media, telecommunication, and power engineering. Yield management models are mostly based on marketing theory, statistics, econometrics, operations research, and financial management.

Yield management technologies for air transport are based on the data on booking and sales of seats provided by computer reservation systems (CRS) of carriers, also known as inventory systems. Inventory systems are storages of databases of airline resources (reference data, schedules, fares, and terms of their use) and management tools for these resources (access to resources, introduction of changes to schedules and fares, setting booking horizons, the booking and sale of seats, etc.).

Yield management technologies are implemented in corresponding automated revenue management systems (ARMS). Based on the flight load data obtained from the airline’s inventory system, ARMS analyze the passenger traffic distribution by flights, routes, and dates. ARMS construct a model and predict the airline’s flight load with specification by booking class codes, flights, and dates. It becomes possible for the airline’s analysts to predict the commercial flight load with the developing market conditions taken into account and various events occurring due to objective reasons (holidays, conferences, emergencies, etc.) considered. It is generally accepted that airlines equipped with such a powerful tool gain certain advantages in the transport market.

In order to optimize revenue from sales of seats, two types of resources are used by airlines: fares and quotas of seats arranged for them. It is implemented within the yield management paradigm by means of ProfitLine/Yield Rembrandt, ProfitLine/Yield O&D Forecaster, and ProfitLine/Yield O&D Optimizer subsystems.

Overbooking is another very efficient tool of the increasing revenue of airlines. It is a marketing policy according to which the booking and sale of seats (with actual payment) is carried out in excess of the available seats on flights (vacancies in hotels, cars for rent, and etc.). Overbooking for flights not only implies booking of seats when no seats are actually available but also the sale of seats that don’t exist with respec-
tive payments. The overbooking practice in air transport is different from the one in, for instance, the hotel industry, where the client guarantees payment for the hotel using her payment card but no money is withdrawn from the card when no vacant room is available. Thereby, overbooking for air flights always implies overselling. These terms will be used as synonyms below.

Overbooking as a marketing policy within a more general yield management strategy first appeared in the US air transport industry in the middle of the 20th century [4].

The strategy relies upon the fact that a lot of passengers don’t show up for flights without notice. The reasons are as follows. First of all, it makes no sense for a passenger to return the cheapest tickets as their cost is not refunded. Second, return tickets may sometimes be cheaper than one-way tickets, which results in one of the flights not being used by the passenger. Third, expensive tickets may in most cases be rescheduled with small fines even after the takeoff. As a result, up to 10 percent of passengers who bought tickets for flights don’t show up for boarding.

In order to minimize losses and/or gain extra revenue, the overbooking strategy was created, which implied submitting more tickets for sale than there were seats on a flight. The strategy was based on the fact that a number of passengers won’t show up for the respective flight. The carrier accepted the overbooking risks, when the number of passengers who arrived at the airport exceeded the number of seats. The carrier mitigated the passenger’s inconvenience by providing her a seat on the next flight of its own or another carrier. When this next flight was to take place in a significant time, the passenger received meals, accommodation, telephone communication with relatives, a monetary refund, etc. at the expense of the carrier.

The optimal overbooking level is set for an airline similarly to the calculations carried out in yield management based on the balance of two components:
— the yield lost due to empty seats on the flight;
— the size of the financial compensation to the passengers who didn’t get on the flight and the reputation losses.

To estimate this balance, the following approach is used [4]. The designations are as follows:

$L$ is the number of passengers who did not show up for the flight;

$F(x) = P\{L \leq x\}$ is the distribution function of the random variable $L$ at the moment of the departure;

$r$ is the number of overbooked seats;

$n^*$ is the number of seats on the aircraft;

$n^* + r$ is the number of seats being sold for the flight with overbooking taken into account;

$a$ represents the airline’s losses per each empty seat on the flight (the ticket price for a seat of the corresponding booking class);

$d$ represents the company’s losses from the compensation of the passenger who did not take the flight as a result of the overbooking strategy.

If $L > r$, then we have $r$ seats sold in excess, and all the available passengers (including the ones due to overbooking) will take the flight.

If $L < r$, then the number of seats sold in excess for the flight is $r - L$. Therefore, $r - L$ passengers won’t take the flight and their losses are to be refunded. The monetary losses caused by each passenger are $d$ in this case.

The authors of [4] suggest obtaining the optimal number of overbooked seats $r^*$ as a result of rounding up the value $r^*$, which is as follows:

$$F(r^*) = \frac{a}{a + d}. \quad (1)$$

This formula seems to be validated as follows. Given the stated conditions, the airline’s losses due to empty seats will be $a(L - r)$ only if $L > r$. The losses due to compensations for passengers with tickets who did not take the flight are $d(r - L)$ only if $L < r$. The event $L < r$ has a probability $F(r) = P\{L \text{ sold } \leq r\}$, whereas the complementary event $L > r$ has a probability $1 - F(r)$. It will be accepted that the average losses will be minimized if the average losses due to the corresponding reasons are equal for one hypothetical passenger. Then, the following equation is available in order to determine the optimal $r^*$:

$$a(1 - F(r^*)) = dF(r^*),$$

from which the presented formula follows. Its formal validation is presented in the Appendix (at the end of the paper).

The explicit expression for the optimal $r^*$ is represented by the formula