INTRODUCTION

Mathematical decision theory is developing rapidly and is gaining greater practical importance. The theory focuses for the most part on single-choice problems, i.e., the problems of selecting one of the best or an optimal object [for example, 1 and 2] from a prescribed aggregate of objects (choices, plans, strategies, or alternatives). If the preferences are simulated with a disconnected binary relation, a strict partial order, then the applicants for the optimal object are non-dominated objects. This is typical, for example, of multiobjective problems when the preferences of the person making decisions (PMD) are not clearly recognized [3].

However, other formulations of the decision problems are often observed. These are the problems of the selection of a few (the chosen number is \(l > 1\)) optimal objects from a prescribed finite aggregate. These problems can be divided into two groups. One of them is the problems in which initially some optimal objects should be selected and then one of the best should be chosen from the selected objects. The problems of creating complicated systems serve as examples: at the first design stage two or three of the best choices are selected from some system alternatives and then after more careful consideration one of the best is chosen between them. These can be called two-stage single-choice problems. Here only non-dominated objects can pretend to be the best.

The other group is multiple-choice problems or subset-choice problems where the application of all \(l\)-selected optimal objects, not excepting any, is supposed. Examples of these problems include competitions and tenders of all types, member group formation, etc.

If a preference relation is given by the aggregate of all \(l\)-selected sets to solve the problem of \(l\)-selected optimal objects. A number of works [4–6] have been devoted to the problem of the preference relation expansion into a finite aggregate to the relation over a power set and its value function construction. However, they supposed that a complete ordering should be expanded into a power set, as well, a number of additional sufficiently strong assumptions, which were formulated as axioms concerning some properties of a desired connected relation over the power set, were taken. Our objective is to expand the partial ordering and only into an aggregate of the sets of \(l\)-objects where any additional assumptions are not taken.

In the literature two approaches to selecting applicants for the best objects in these problems (for example, [7]) were considered. According to the former all non-dominated objects of “the first level” should be taken. If the objects are less than \(l\), the second “level” formed by non-dominated objects in the aggregate remaining after a deletion of the first “level” objects from an initial aggregate should be added to them. If all objects are less than \(l\), the third “level” objects should be added to them and etc. until for the first time the number of all chosen objects are not less \(l\). The latter approach regards that all \(l\)-such “levels” should be taken. But in [8] it was shown that the former approach narrows unreasonably the number of applicants for the best objects and the latter in contrast, expands it unreasonably. A corresponding example will be given in this paper.

In [8], the concept generalization of a non-dominated object was introduced for multiple-choice problems: an object is called \(l\)-nondominated if no more than \(l - 1\) objects are more preferred than it in accordance with a strict order relation. Also, it was found that only \(l\)-nondominated objects can pretend to be \(l\)-optimal. In [9], a number of the main properties of \(l\)-nondominated objects was presented, the formulas in...
order to calculate the performance evaluations of the decision rules in the multiple-choice problems was given and the Pareto’s decision rule performance was evaluated. In [10], the performance evaluations of the decision rules using information on the importance of the homogeneous tests were obtained [11]. In [12, 13] the authors give the collection of both previously obtained and recent results in the performance evaluations of some decision rules as applied to the various decision problem formulations, including the multiple-choice problems and the rule using information on the relative importance of the homogeneous tests [8, 14]. In [15], it was shown that the development level of the theory under discussion is clearly inadequate to meet demands in practice and the direction for future development was defined.

In this paper the problem of selecting a few optimal objects is examined within the framework of the general selection methodology when the preference relations are partial, the procedure for forming the set of selection methodology when the preference relations are partial, the procedure for forming the set of optimal objects under consideration is examined. As well, in (3) both the equivalence and the indifference relation result from (1) to (3) being performed as :\( = \). Finally, if there are not elements then \( = \) is performed as \( = \).}

It should be noted that Definition 1 in its own way is relative to the definition of the preference—indifference relation of the multiobject problems with equally important homogeneous tests [18].

The relation \( R' \), as is easy to understand, is a quasiorder. It leads to the strict preference relation, the strict partial order \( P \) and the indifference relation, the equivalency \( I \) in \( L \). It is clear that \( A R' B \) is performed when in (1) and (2) \( R \) can be substituted by \( I \) for each \( i \) and \( A P' B \) is correct if \( R \) can be replaced by \( P \) only for a single \( i \) in (1) and (2).

**Definition 1.** The relation \( A R' B \) is performed if and only if these permutations \( \pi \) and \( \pi' \) exist that are true:

\[
a^{(i)}_{\pi(i)} Rb^{(i)}_{\pi'(i)}, \quad i = 1, ..., l.
\]  

(1)

It is easy to understand that Definition 1 is equal to either of the two following definitions:

\[
AR' \iff \exists \pi \in \Pi: a^{(i)}_{\pi(i)} Rb^{(i)}_{\pi'(i)}, \quad i = 1, ..., l; \\
AR' \iff \exists \pi \in \Pi: a^{(i)}_{\pi(i)} Rb^{(i)}_{\pi'(i)}, \quad i = 1, ..., l.
\]  

(2)

Let \( P'(L), R'(L), \) and \( \bar{P}'(L) \) be the sets of strictly optimal, optimal, and nondominated sets respectively. By Definitions 1 and 2 the relations

\[
P'(L) \subseteq R'(L) \subseteq \bar{P}'(L),
\]  

(3)

are true. If there is an optimal set then it exhausts the set \( P(L) \) obviously \( [A R' B = P(L) \to R'(L) = P'(L)] \) and the desired \( I \) of the best objects are defined univalently — these are all objects from the set. As well, in (3) both \( \subseteq \) are performed as the equalities \( = \). Suppose that this set does not exist but there is an optimal set. Even if it is not single, all optimal sets are equivalent to the relation \( I \). Therefore, any of them can have “equal right” to pretend to be the best and its forming objects can be regarded as \( l \)-best. In addition, the second side (right side) \( \subseteq \) in 3 is performed as \( = \). Finally, if there are not any optimal objects, by finiteness of the set \( L \), nondominated sets will be certain to exist. Moreover, the set of all sets will be outwardly stable: for any set \( B \in L \setminus \bar{P}'(L) \) the set \( A \in \bar{P}'(L) \) will be found so that \( A P' B \) is correct (see Theorem 2, given below). Therefore, the applicants for the optimal set can be only nondominated sets and the objects from these sets can pretend to be \( l \)-best.

Unfortunately, in the last case the set \( \bar{P}'(L) \) will contain noncompared sets in \( R' \). Such cases can be