To date, an absolute majority of astronomical observations have been made through the terrestrial atmosphere with characteristic random inhomogeneities of refraction index. Their influence will cause a decrease of the telescope resolution in comparison with its diffraction limit. Many researchers worked on this problem, trying to overcome the interfering influence of atmosphere. A lot of methods were proposed to diminish this influence, being very effective in some cases. Here, interferometer of Fizeau-Michelson [22], implementation of restoration filter [4, 16, 21], speckle interferometry [20], use of a telescope with non-redundant aperture [23], multiple-beam interferometer (development of Michelson’s idea) [6, 11, 24], and adaptive optics [1, 17] should be mentioned. Some of these methods need the application of special optical devices, replacing or supplementing the traditional telescope. However, speaking about observations by means of a traditional telescope, one should remember that the problem of optimal signal accumulation in this case is yet not solved, thus leaving a hope that possibilities of traditional telescope are not exhausted [9]. This stimulates attempts aiming at improving the signal accumulation under such observations, and our work is devoted to one of these attempts.

The atmospheric inhomogeneities distort randomly both modules and phases of the Fourier-components of the image of an object. The Labeyrie method (method of speckle interferometry) makes it possible to effectively accumulate the Fourier-component modules by summing their squared values on a sequence of shots. However, in this case, information about their phases turns out to be lost. Therefore, there is an idea to improve the Labeyrie method by supplementing it with a procedure that allows for accumulating the phases in some way. Similar ideas were proposed in [14, 19], but studies in this direction were not developed at that time. Now, it is time to discuss these ideas once again and to examine their possibilities in detail. This work is the first step in this direction.

INFLUENCE OF ATMOSPHERE ON ASTRONOMICAL IMAGE

From the viewpoint of wave optics, a possibility to image a remote object by a telescope is based on the fact that distribution of complex amplitude of monochromatic coherent light in the back focal plane of a lens is the Fourier transform of the complex amplitude of the field in front of focal plane [12]. When an object is observed in noncoherent light, only the time average of light intensity is accessible at each point of the back focal plane, because the amplitude changes too quickly. In case of infinite aperture, the dependence of the intensity on coordinates is expressed by a function that is the Fourier transform of the function of wave field coherence coming from the object. Since the coherence function, according to van Zittert-Zernike theorem [2], is the Fourier transform of an object brightness, the brightness in the back focal plane proves to be the Fourier transform of the object brightness Fourier transform; i.e., it is its turned-over image. In a real case, at finite aperture, this image turns out to be a convolution of the actual image \( I_0(x, y) \) with the telescope diffraction kernel \( G_0(x, y) \) representing the Fourier transform of the space-frequency characteristic of a telescope without atmospheric influence.

\[
\tilde{G}_0(k_x, k_y) = \int \int a^*(k_x' - k_x, k_y' - k_y) a(k_x', k_y') dk_x' dk_y',
\]

(1)
where \( a(\xi, \eta) \) is an aperture function, which equals the unit within the aperture and zero out of it. The width of this kernel determines the diffraction limit of the telescope, being inversely proportional to a diameter of aperture.

When observing an object through a thin layer of a medium with small inhomogeneities of refraction index, being localized close to an observer, one can consider that only a phase of the incoming wave is subjected to a distortion. This distortion has a random nature, and it depends on coordinates \( \xi \) and \( \eta \) in the focal plane of a telescope and on time \( t \). It is described by an \( \text{a priori} \) unknown function \( \delta(\xi, \eta, t) \). Due to this distortion, the image formed by the telescope proves to be a convolution of the actual image with the kernel \( G(x, y, t) \), being the Fourier transform of the space-frequency characteristic of the telescope in the presence of atmosphere

\[
\tilde{G}(k_x, k_y, t) = \int \mathcal{A}^+(k_x - k'_x, k'_y - k_y, t)A(k'_x, k'_y, t)dk'_xdk'_y,
\]

where \( A(\xi, \eta, t) \) is the atmosphere-aperture function that equals \( M(\xi, \eta, t)\exp[i\delta(\xi, \eta, t)] \) within the aperture and zero out of it. For a thin atmospheric layer close to a telescope, one can assume \( M(\xi, \eta, t) = 1 \). When an object is imaged with a long exposure time, the obtained image is given by a convolution of \( I_0(x, y, t) \) with the kernel \( g(x, y, t) \), which is given by integral of \( G(x, y, t) \) taken over the time of exposition. The influence of atmosphere at a long exposition time suppresses the high space frequencies in an image and considerably diminishes the telescope resolution, often by tens times and more.

The properties of instantaneous space-frequency characteristic (2) depend on statistics of phase distortions of a wave produced by inhomogeneities of refraction index in atmosphere. There are various concepts concerning these statistics. In some researches, it is assumed that the field of phase distortions is represented by Kolmogorov’s process [15] with the structural function \( r^{2/3} \), where \( r \) is the distance between the points in plane of aperture. In this case, the scale of inhomogeneities is characterized by Fried’s parameter [18]. In particular, such a hypothesis is accepted in [10]. Below, we assume that the field of phase distortions is described by stationary Gaussian process with spectral density

\[
W(k) = a\exp(-\beta k^2).
\]

This hypothesis is favored by the results of analysis of the phase distortion field during night observations with account for thermal conductivity of air. This question is considered below in more detail.

In practice, it is convenient to use an effective size of inhomogeneities

\[
l \sim 2\sqrt{\beta}.
\]

Atmosphere influence on image largely depends upon parameter \( D/l \), where \( D \) is a diameter of the telescope aperture. In any case, the simultaneous atmospheric distortion can be conditionally separated on trembling (random shift of an image as a whole) and blurring can be a convolution of the nonshifted image with the distorting kernel located at an origin. At fixed \( l \), an average rms angle of trembling is proportional to the rms value of the phase perturbation \( q \). At \( D < 1 \), in case of short exposition, trembling dominates blurring, whereas, in the case of long exposition, the blurring proves to be the main factor of distortion, corresponding to Gaussian kernel. At \( D > 1 \), the main distortion effect is related with the instantaneous atmosphere-aperture kernel, and trembling is small. The lack of significant trembling under usual observations made with large telescopes confirms that the field of phase distortions is closer to a stationary Gaussian process at favorable astronomical conditions. Certainly, the statistics of atmospheric distortions depends on the place and time of these observations.

Bearing in mind Eq. (2), in the absence of noise, the influence of atmosphere on image can be considered as the random noise of multiplicative form in the frequency representation. The modulus \( M \) of the noise Fourier component has a narrow distribution near \( M = 1 \), whereas its phase is described by the Gaussian random variable with zero average value and dispersion depending on \( q \). For the present moment, the correlation properties of this noise (in Fourier plane) are out of scope of our interest.

ASSUMPTIONS ON STATISTICAL PROPERTIES OF THE PHASE DISTORTION FIELD

As was already noted, degree and character of distortion of an astronomical object depend on statistical properties of the phase distortion field. It is generally agreed that atmospheric inhomogeneities are a stationary turbulence. To a greater extent, this is conditioned more by a well-developed theory of stationary turbulence than by experimental evidence [5]. As follows from this theory, \( \delta(\xi, \eta, t) \) is a realization of Kolmogorov’s process. There are, however, some objections against this picture, and the most serious of them are given below.