BASIC ASSUMPTIONS

(1) A sample manufactured from uniform and isotropic material is examined.

(2) Initially, the sample has a cracklike defect. The defect tip is blunted with the radius of the curvature, which is significantly less than the sample sizes and, therefore, the sizes of the plastic areas joined to concentrators’ tips are significantly less than the sizes of the elastic areas of the sample.

(3) At infinity, the sample is loaded by cyclic stresses orientated normally to the crack direction. The maximal and minimal stresses of the loading cycle are $\sigma_{\text{max}}$ and $\sigma_{\text{min}}$, respectively. In this case, the magnitude of the stresses is less than the material yield point.

(4) The cyclic loading of the sample at each semicycle of the external load is simulated by the serial application of equal tensile and compressive stresses ($\sigma_{\text{max}} - \sigma_{\text{min}}$). We investigate how the stressed-deformed state of the defect tip varies if the external load is changed from $\sigma_{\text{min}}$ up to $\sigma_{\text{max}}$ during loading, and from $\sigma_{\text{max}}$ down to $\sigma_{\text{min}}$ during unloading.

(5) During the process of straining the sample, the deformations at any point can be higher than the limit value $\varepsilon_B$ (the material fails at the moment when the deformation becomes higher than $\varepsilon_B$); under compression, the deformation value is not limited.

(6) The steady state failure process is investigated. The crack geometry and stressed-deformed state of the material in the defect tip were formed during the previous long-term cyclic loading of the sample. The mechanisms of crack initiation and brittle fracture are not studied.

ELASTIC–PLASTIC DEFORMATION OF HARDENING MATERIALS

Let the sample material, in which there is a crack, be a hardening material. Let us present its diagram of deformation as a piecewise continuous function that consists of a straight line, which describes the elastic segment with the modulus of elasticity $E$, and a power function that describes the elastic–plastic deformation of the material: $\sigma(\varepsilon) = E\varepsilon$: the initial segment is described by Hooke’s law and the hardening segment [1] is presented as follows:

$$\sigma(\varepsilon) = \sigma_T \left( \frac{\varepsilon}{\varepsilon_T} \right)^n, \quad 0 \leq n \leq 1,$$

where $\sigma_T$, $\varepsilon_T$ are the yield point and the yield strain of the sample.

Such a presentation of the stress–strain diagram makes it possible to transform the diagram into the particular cases:

(1) $n = 0$. A perfectly plastic material. Under $\varepsilon \geq \varepsilon_T$, we have $\sigma(\varepsilon) = \sigma_T$, i.e., we obtain the classical Prandtl diagram.

(2) $n = 1$. Absolutely brittle material, which is described by Hooke’s law in the whole range of loading until total failure.

Let the cracklike defect have a length of $2l$ and the radius of the curvature in the tip be $\rho_1$. During direct loading (if the value of external stresses rises up to $\sigma_{\text{max}}$), the deformation at the defect tip grows and its
value can be higher than the yield point. Let us compare the level of external stresses and the value of the plastic deformation at the crack tip. For this purpose, let us write the Neiberg relation [2] for the coefficients of the concentration

\[ k_{\text{elast}}^2 = k_{\sigma} k_{\varepsilon}, \]  

(2)

where \( k_{\text{elast}} = \frac{\sigma_{\text{elast}}}{\sigma_{\text{rated}}}; k_{\sigma} = \frac{\sigma}{\sigma_{\text{rated}}}; k_{\varepsilon} = \frac{\varepsilon}{\varepsilon_{\text{rated}}}; k_{\sigma}, k_{\varepsilon} \) are the concentration coefficients of stresses and deformations calculated at the tip of the cracklike defect; \( \sigma_{\text{rated}} \) and \( \varepsilon_{\text{rated}} \) are the elastic stresses and deformations that act in the undisturbed area of the sample, and they are connected by the Hooke’s law: \( \sigma_{\text{rated}} = E\varepsilon_{\text{rated}}; \sigma, \varepsilon \) are the stresses and elastic–plastic deformations acting in the crack tip, respectively; and \( \sigma_{\text{elast}} \) are the stresses that would act in the concentrator tip under the assumption that the sample material is described by Hooke’s law within the whole range of loading until failure.

In this case, Eq. (2), by taking into account Eq. (1), is transformed as follows:

\[ \frac{\varepsilon}{\varepsilon_T} = \left( \frac{\sigma_{\text{elast}}}{\sigma_T} \right)^{\frac{2}{n+1}}. \]  

(3)

As it is for the case of fatigue crack propagation in perfectly plastic material [3], we use the following set of hypotheses in the present paper:

(1) The sample material is characterized by translational hardening properties. During repeated cyclic loading, the modulus of elasticity and the yield point are constant.

(2) The plastic area at the defect tip is deformed from the side of the elastic part of the material according to the law of stiff loading.

(3) The points at the crack edges move strictly along the direction of the external load, i.e., normally to the major semiaxis of the ellipsoidal crack.

Let the sample material be characterized by the stress–strain diagram, which is described by power dependence (1), which has a loop under cyclic loading as is shown in Fig. 1. We investigate the deformation increment under cyclic loading by external stresses at each semicycle by the value of \( \sigma_{\text{max}} - \sigma_{\text{min}} = \sigma_{\text{max}}(1 - r) \), where \( r = (\sigma_{\text{min}}/\sigma_{\text{max}}) \) is the coefficient of the asymmetry of the loading cycle. In this case, the elastic stresses acting at the tip of the cracklike defect can be written as follows: \( \sigma_{\text{elast}} = k_{\text{elast}}\sigma_{\text{max}}(1 - r) - \sigma_T \).

This equality, by considering the known Neiberg representation [4] for the coefficient of the stresses concentration \( k_{\text{elast}} = 1 + 2\sqrt{l/\rho_1} \) and by the fact that \( l \gg \rho_1 \), can be presented as follows:

\[ \frac{\sigma_{\text{elast}}}{\sigma_T} = 2 \sqrt{\frac{l}{\rho_1}} \frac{\sigma_{\text{max}}(1 - r)}{\sigma_T} - 1. \]

Further, when we examine the variation of the stressed-deformed state of the crack tip, we introduce the coordinate system \( \tilde{\sigma} - \tilde{\varepsilon} \) (Fig. 1). If we take into account the Bauschinger effect, its origin of the coor-