On the Self-Synchronization of Inertial Vibration Exciters with an Internal Degree of Freedom

M. A. Potapenko

Institute of Problems of Mechanical Engineering, Russian Academy of Sciences, St. Petersburg
e-mail: m.potap@mail.ru
Received December 17, 2014

Abstract—This paper considers a model of the behavior of two coaxial unbalanced vibration exciters, with an internal degree of freedom, mounted on a solid, and three conditions to ensure the timing stability of the common mode. An analytic expression for the radius of the movable mass is found, wherein Blekhman–Sperling stability conditions are expressed through system parameters; regions of changes in the parameters for which all the conditions are met are established.

DOI: 10.3103/S1052618815040111

To date, the theory of synchronization of rotating objects without internal degrees of freedom has been fairly well developed. On the other hand, rotors with an internal degree of freedom referred to in [1] for the first time were considered in the simplest system. Later, in [2], the problem for the general case of synchronization of rotating bodies with internal degrees of freedom was set. In this study, we obtained equations of motion for a system consisting of two identical coaxial vibration exciters centrally mounted on a flat vibrating solid and each having two additional degrees of freedom. A special feature of this case is the possibility of obtaining an exact solution for the stationary synchronous mode and studying its stability in the small. It was found that through additional degrees of freedom of the rotors, the common mode of rotation, which is unstable in the post-resonance field, can be sustained in the small under three conditions limiting system parameters. However, this stability is temporary (or gyroscopic). The obtained conditions contained an intermediate value, the radius of the movable mass in a steady state.

The purpose of this paper is to analyze the results of [2], in particular, the received conditions of stability of the synchronous and common mode of rotation of rotors (Blekhman–Sperling conditions). It is shown that the satisfaction of two conditions under normal practice parameter values guarantees the implementation of the third, and all the three conditions are satisfied in a narrow range of variation of rotor speed.

Specific devices that correspond to the model under consideration are, for example, a vibration mill and an apparatus for the vibration processing of components [3].

System description. The scheme of a vibrating device with rotors having internal degrees of freedom is shown in the figure, where a carrier $I$ in the form of a solid body mounted on springs with known characteristics and performing plane motion. Two coaxial debalances $2$ are rigidly fixed on it.) Within each debalance additional mass $3$ is fixed on a spring. This mass can oscillate in the radial direction. In contrast to [2], it is assumed that the values of the stiffness of the support springs ($C_x$) and ($C_y$) are finite. The system is considered to be balanced, so there are no rotary oscillations of body $I$.

Thus, the position of the carrier is determined by the coordinates $x$ and $y$ of each $i$th debalance, for one rotational $\phi$, and one translational coordinate defining the distance $\rho_i$ of the additional mass from the axis of the rotor.

The system of equations describing the behavior of the vibration device with internal degrees of freedom of unbalanced rotors. In [2], we obtained the equations of motion of the vibration device with rotors having internal degrees of freedom.
described system. For the case where additional masses can only move in a radial direction (figure), the
equations have the form

\[ [I + m_{0}\varepsilon^2 + m(r + \rho_s)^2] \ddot{\phi}_s + K_s(\dot{\phi}_s - \omega) + 2m(r + \rho_s)\dot{\phi}_s \ddot{\phi}_s - [m_{0}\varepsilon^2 + m(r + \rho_s)^2] \]

\times (\ddot{x}\sin\phi_s + \ddot{y}\cos\phi_s) = K_s(\omega_s - \omega), \quad s = 1, 2,

\[ M\ddot{x} = \sum_{i=1}^{2} [m_i\varepsilon + m(r + \rho_i)](\ddot{\phi}_i\sin\phi_i + \dot{\phi}_i\cos\phi_i) - m\sum_{i=1}^{2}\ddot{\phi}_i\cos\phi_i \]

\[ + 2m\sum_{i=1}^{2}\dot{\phi}_i\sin\phi_i - \beta_x\ddot{x} - C_xx, \]

\[ M\ddot{y} = \sum_{i=1}^{2} [m_i\varepsilon + m(r + \rho_i)](\dot{\phi}_i\cos\phi_i + \dot{\phi}_i\sin\phi_i) + m\sum_{i=1}^{2}\ddot{\phi}_i\sin\phi_i \]

\[ + 2m\sum_{i=1}^{2}\dot{\phi}_i\cos\phi_i - \beta_y\ddot{y} - C_yy, \]

\[ \ddot{\phi}_s + \beta_\phi\dot{\phi}_s + \omega^2_\phi\phi_s = (r + \rho_s)\dot{\phi}_s - (\ddot{x}\cos\phi_s - \ddot{y}\sin\phi_s), \]

where \( M = M^0 + 2m_0 + 2m, \omega^2_\rho = C_\rho/m. \)

Here, in contrast to [2], we take into account the terms \( \beta_x\ddot{x} \) and \( \beta_y\ddot{y} \), which take into account the resistance
to fluctuations of the solid body, so that the synchronous frequency \( \omega \) differs from the partial
frequencies \( \omega_1 = \omega_2 = \omega_s \). From a physical standpoint, \( \omega < \omega_s \), as in this case part of the energy consumed
by electric motors is used to overcome the losses from the oscillations of the body 3.

In equations (1), \( M^0 \) is the mass of the carrier; \( m_0 \) is the additional movable mass inside the debalance;
\( m \) is the mass of the device; \( I \) is the moment of inertia of the device; \( r \) is the moment of inertia
of the solid portion of the debalance from its axis of rotation; \( r \) is the moment of inertia
of the movable mass from rest in the direction of increasing radius; \( x \) is the displacement of the center
of mass of the carrier horizontally; \( y \) is the displacement of the center of mass of the carrier vertically;
\( \omega \) is the synchronous angular velocity of the rotor in the steady state; \( \phi \) is the angle of rotation of the rotor
measured clockwise from the horizontal direction; \( \beta_x = \beta_y = \beta \) are the coefficients of the carrier
oscillation damping along the respective axes; \( C_x = C_y = C \) are the stiffness coefficients of the springs on
which the platform is are mounted, along the respective axes; \( \beta_\phi \) is the coefficient of linear resistance to
oscillations of the movable mass along the radial guide; \( C_\rho \) is the stiffness coefficient of the spring inside
the rotor; \( K \) is the total damping coefficient; \( \omega_s \) is the partial rate of the exciter, that is, the angular velocity
reached by each rotor mounted on a fixed base; \( \omega_\rho \) is the partial frequency of the internal mass.

For the stationary common mode of motion of the system with the same exciters \( \phi_1 = \phi_2 = \omega, \phi_1 = \phi_2 = \omega_t, \rho_1 = \rho_2 = \rho^0 = \text{const}, \omega_1 = \omega_2 = \omega_s \) from (1) the following system of equations for determining
the coordinates \( x, y, \rho \) and the angular velocity of synchronous rotation \( \omega \) is obtained:

\[ [m_0\varepsilon + m(r + \rho^0)](\ddot{x}\sin\omega t + \ddot{y}\cos\omega t) = K(\omega - \omega_s), \]

\[ M\ddot{x} = 2[m_0\varepsilon + m(r + \rho^0)]\omega^2\cos\omega t - \beta_x\ddot{x} - C_xx, \]

\[ M\ddot{y} = 2[m_0\varepsilon + m(r + \rho^0)](-\omega^2\sin\omega t) - \beta_y\ddot{y} - C_yy, \]

\[ \omega^2_\rho = \omega^2 - (\ddot{x}\cos\omega t - \ddot{y}\sin\omega t). \]