Longitudinal—Transverse Dynamics of Two-Component Pulses in Uniaxial Crystals

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Abstract—Based on the Ritz—Whitham variational principle, a set of equations is derived that describe the longitudinal—transverse dynamics of parameters of an electromagnetic pulse whose ordinary component is a quasi-monochromatic pulse and whose extraordinary component is a videopulse. The critical parameters that determine the pulse stability relative to self-focusing are determined.

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INTRODUCTION

Anderson [1] proposed using an “averaged Lagrangian” of the Ritz–Whitham type to find approximate solutions of nonlinear equations. In this work, we used this approach to investigate the dynamics of quasi-monochromatic pulses, which propagate in isotropic dielectrics and are described by a nonlinear Schrödinger equation (NSE). It is shown that our NSE has a soliton solution in the region of anomalous dispersion of group velocity, while in the region of normal dispersion of the group velocity, it describes the generation of a spectral supercontinuum [2], manifested in the broadening of the signal.

Sazonov and Sobolevskii [3] derived a set of equations that describes the dynamics of a two-component pulse propagating at an arbitrary angle to the optical axis of a uniaxial crystal in its region of transparency. It was also shown that this system is transformed into an NSE with pulse propagation along the optical axis and into the Yajima–Oikawa system, the soliton solution for which is well known, with propagation perpendicular to the optical axis [4]. This solution describes the bound state of the quasi-monochromatic pulse of the ordinary component and the videopulse of the extraordinary component; the extraordinary component is generated by the ordinary one during signal propagation. The logical question of finding the general solution (allowing for transverse dynamics), which would include both limiting cases, then arises.

THE RITZ–WHITHAM VARIATIONAL PRINCIPLE AND THE SET OF EQUATIONS FOR PULSE PARAMETERS

The above set, which describes the dynamics of an ordinary—extraordinary pulse in a uniaxial crystal, has the form [3]

\[ \frac{\partial E_o}{\partial z} + \frac{n_o \partial E_o}{c \partial t} + a_2 \frac{\partial}{\partial t} (E_o E_o) + a_3 \frac{\partial}{\partial t} (E_o^2 E_o) + b_2n_o E_o \frac{\partial E_o}{\partial t} - \delta_o \frac{\partial^3 E_o}{\partial t^3} + \sigma \int E_o dt' = \frac{c}{2n_o} \Delta_z \int E_o dt', \]

\[ \frac{\partial E_e}{\partial z} + \frac{n_e \partial E_e}{c \partial t} + a_1 E_e \frac{\partial E_o}{\partial t} + b_2 E_e \frac{\partial E_o}{\partial t} + a_3 \frac{\partial (E_o^3 E_e)}{\partial t} + b_3 E_e \frac{\partial E_o}{\partial t} - \delta_e \frac{\partial^3 E_e}{\partial t^3} + \sigma \int E_e dt' = \frac{c}{2n_e} \Delta_z \int E_e dt'. \]

Here, \( E_o, E_e \) is the electric field of the ordinary and extraordinary components; \( z \) is the axis along which the pulse propagates; \( c \) is the speed of light in a vacuum; \( n_o, n_e \) represents ordinary and extraordinary refractivity; coefficients \( a_2, b_2, a_3, b_3 \) determine the contributions from nonlinearities of the second and third orders; parameters \( \delta_o, \delta_e \) characterize electronic dispersion; and parameter \( \sigma \) characterizes the ionic dispersion. It can be seen from set (1), (2) that the ordinary component can generate an extraordinary component, due to the nonlinear summand at \( a_3 \). It was shown in [3] that the intensity of the generated ordinary component is lower than the initial signal by several orders of magnitude; we therefore ignore the intrinsic nonlinearity and dispersion of the extraordi-
First, we ignore cross-cubic nonlinearities but not quadratic ones. In this work, we consider a case in which the ordinary component is a quasi-monochromatic pulse. In this case, the electric field can be represented using envelope $\psi$ in the form

$$E_0 = \psi e^{i(\omega r - k z)} + c.c.,$$

where $k$ is the wave number and $\omega$ is the band center of the ordinary component.

Inserting (3) into (1), (2) and using the above approximations, we derive

$$i \frac{\partial \psi}{\partial t} + \beta \frac{\partial^2 \psi}{\partial t^2} + \alpha \psi |\psi|^2 = a_2 \omega E_0 \psi + \frac{c}{2n_e \omega} \frac{\partial^2 \psi}{\partial x^2},$$

(4)

$$\frac{\partial E_0}{\partial t} = -a_2 \frac{\partial}{\partial t} |\psi|^2 + \frac{c}{2n_e} \frac{\partial^2}{\partial x^2} \int E_0 d\tau.$$  

(5)

Here, $\beta = -3\delta_0 \left( \omega^4 - \omega^4 \right) / \omega^3$, $\omega_0^4 = \sigma / 3 \delta_0$, $\alpha = -b_0 \omega$, and $t = t - z / v_0$ is time in the corresponding system of coordinates; $v_0 = n_0 / c + 3 \delta_0 \omega / \omega_0$ is the pulse group velocity. In deriving (4), (5), we included the Zakharov–Benney resonance condition, according to which the group velocity of the ordinary (high-frequency component) equals the phase velocity (low-frequency) of the extraordinary component. This condition can be written in the form

$$\frac{n_e}{c} = \frac{n_0}{c} + 3 \delta_0 \omega_0^2 + \frac{\sigma}{\omega_0^2}.$$  

(6)

If we consider the one-dimensional case (not allowing for the transverse signal dynamics), Eq. (4) is transformed into an NSE with pulse propagation along optical axis $a_2 = 0$. If the pulse propagates perpendicular to the optical axis, the quadratic nonlinearity is maximal; if we ignore summands at $a_2$ and $\alpha$ were proportional to $\exp(-x^2 / 2 \rho^2)$.

Using the above and considering that the pulse has the Gaussian transverse profile, we select trial solutions for pulse propagation at an arbitrary angle to the optical axis:

$$\psi = A \text{sech} \left[ (\tau - \Phi)/a \right] \exp \left[ i (\varphi - \Omega (\tau - \Phi) + \epsilon (\tau - \Phi)^2 + G x^2 - \varphi^2 / 2 \rho^2 \right],$$

(7)

$$U = -B \tanh \left[ (\tau - \Phi)/a \right] \exp \left( -\varphi^2 / 2 \rho^2 \right).$$

(8)

Here, $E_0 = \partial U / \partial \tau$, $A$, $B$ are proportional to amplitudes of the ordinary and extraordinary components; $\epsilon$ describes the frequency modulation of the ordinary component; $\Omega$ determines the magnitude of the shift of the band center; $G$ characterizes the curvature of the phase wave surfaces of the input pulse; $\varphi$ and $\Phi$ determine the contributions to the phase and group velocities; $a$ is the pulse duration, and $\rho$ is its transverse width. We consider that all of these quantities depend on coordinate $z$.

The Lagrangian of set (4), (5) has the form

$$L = \frac{i}{2} \left( \psi^2 \frac{\partial \psi}{\partial t} - \psi \frac{\partial^2 \psi}{\partial t^2} - \beta \frac{\partial^2 \psi}{\partial x^2} - a_2 \omega |\psi|^2 \frac{\partial U}{\partial t} \right) + \frac{\alpha}{2} |\psi|^4 - \frac{\omega_0}{2} \frac{\partial U}{\partial \tau} \frac{\partial \psi}{\partial x} + c \frac{\partial^2 \psi}{\partial x^2} \left( \frac{\partial U}{\partial x} \right)^2.$$  

(9)

Inserting (7) and (8) into (9) and integrating over time and coordinate, we find the averaged Lagrangian

$$\Lambda = \int \int L dx d\tau = 2 \sqrt{\pi} \left[ -a_2 A^2 \left( \frac{1}{3a} + \Omega^2 a + \frac{\pi^2 a^3}{3} \right) \right] / 12$$

(10)

Here and below, the solidus denotes the derivative with respect to $z$. In deriving (10), we considered that without allowing for transverse dynamics, the averaged Lagrangian (10) should transform into the one that corresponds to the set of one-dimensional equations in [5]. We therefore considered that nonlinear summands at $a_2$ and $\alpha$ were proportional to $\exp(-x^2 / 2 \rho^2)$.

Varying (10) over $A$, $B$, $\epsilon$, $\Omega$, $\varphi$, $\Phi$, $a$, and $\rho$ and transforming the derived set, we obtain

$$A^2 a_0 = \text{const} = P,$$  

(11)

$$\Phi' = -2 \beta \Omega,$$  

(12)

$$B = \frac{a_2 a A^2}{2 \beta \Omega},$$  

(13)

$$\epsilon = -\frac{d}{4 \beta a},$$  

(14)

$$G = -\frac{d^2 n_0 \omega_0}{2 \epsilon \rho},$$  

(15)

$$\varphi' = \beta \left( \Omega^2 - \frac{2 \beta}{3 \sigma^2} + \alpha A^2 + \frac{a_2 \omega A^2}{2 \beta \Omega} + \frac{c}{2n_0 a_0 \rho} \right),$$  

(16)

$$\left( \Omega - \frac{\omega P}{3 \sigma^2} \right) \left( \frac{d_2}{2 \rho} \right)^2 \left( \frac{1}{a_0 \Omega^2} \right) = 0,$$  

(17)

$$a'' = \frac{4 \beta}{\pi^2} \left( \frac{4 \beta}{a_0^2} + \frac{a_2 \omega P}{a_0^2} - \frac{2 a_2 P}{a_0^2} \right),$$  

(18)

$$\rho'' = \frac{2 c}{a_0 n_0 a_0 \rho} \left( \frac{a_2 \omega P}{a_0^2} + \frac{a_0 P}{3} + a_2 P \right).$$  

(19)

Set (11)–(19) describes the longitudinal–transverse dynamics of a pulse propagating at an arbitrary