Possibility of the Charge Density Modulation of an Electron Bunch in the Electromagnetic Wave Field

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Received February 21, 2007

Abstract—We consider the problem of modulation of the electron density in a linear bunch after its interaction with a linearly polarized, monochromatic electromagnetic wave, as well as an opportunity to observe this effect. It is shown that one can achieve significant values of the modulation depth at high intensities of the electromagnetic wave.

PACS numbers: 41.85.Ew

DOI: 10.3103/S1068337207040017

Key words: electron bunch, electromagnetic wave

1. INTRODUCTION

Obtaining of dense electron bunches of ultrashort length is a very urgent problem in creation of free-electron lasers, in particular, in the terahertz frequency range. One of possible ways to achieve this goal is the modulation of density of a relatively long bunch, when it interacts with a strong electromagnetic wave of a linearly polarized laser beam and when a sufficiently deep modulation of the charge density distribution can serve as a good basis for the fission of the bunch (realization of the chopping (slicing)).

In [1] the effect of scanning of a linear electron bunch in the field of an intense plane monochromatic wave was studied. It was shown that one can obtain the transverse expansion of the electron bunch and determine its density by means of its scanned image in the plane transverse to the motion. In the present paper we study the charge density variations in a bunch along its length after its interaction with a highly intense, linearly polarized, plane monochromatic wave (laser field).

2. ALGORITHM OF MODULATION OBSERVATION

A setup, which is proposed to study the interaction of an electron bunch with a monochromatic electromagnetic wave, coincides in principle with a device proposed in [1].

The electromagnetic field is chosen to be polarized along the y-axis:

\[ E_y = H_z = E_0 \cos(kx - \omega t + \varphi_0). \]  

(1)

Substituting the expressions for fields into the equations of motion, we obtain

\[ \frac{dx}{dt} = \frac{p_x}{\sqrt{m_0^2 c^2 + p_x^2 + p_y^2}}, \quad \frac{dy}{dt} = \frac{p_y}{\sqrt{m_0^2 c^2 + p_x^2 + p_y^2}}, \quad \frac{dp_x}{dt} = eE_y \frac{p_y}{\sqrt{m_0^2 c^2 + p_x^2 + p_y^2}}, \quad \frac{dp_y}{dt} = eE_y \left(1 - \frac{p_x}{\sqrt{m_0^2 c^2 + p_x^2 + p_y^2}}\right). \]  

(2)

Introducing the parameter \( \eta = \omega t - kx \) and then passing from the time differentiation to the differentiation with respect to \( \eta \) (as it was done in [2, 3]), in order to solve the system of equations (2) and (3), assuming that at the instant of entry of the bunch into the system \( p_{y0} = 0 \) and \( x_0 = 0, \ y_0 = 0 \) (transverse sizes of the bunch are negligibly small), we get

\[ x = \frac{2B}{k} \left[ p_{x0} + \frac{Be^2E_0^2}{2\omega^2} \left(1 + 2\sin^2 \varphi_0 \right) \right] \eta + \frac{Be^2E_0^2}{2k\omega^2} \left[8\sin \varphi_0 \cos(\eta + \varphi_0) - 3\sin 2\varphi_0 - \sin 2(\eta + \varphi_0) \right]. \]  

(4)
\frac{y}{k\omega} = \frac{2BeE_0}{k\omega} \sin \phi_0 \eta - \frac{2BeE_0}{k\omega} \left[ \cos(\eta + \phi_0) - \cos \phi_0 \right], \quad (5)

p_x = p_{so} + \frac{Be^2E_0^2}{\omega^2} \left[ \sin(\eta + \phi_0) - \sin \phi_0 \right]^2, \quad (6)

p_y = \frac{eE_0}{\omega} \left[ \sin(\eta + \phi_0) - \sin \phi_0 \right], \quad (7)

where

\[ B = \sqrt{\frac{m_0^2c^2 + P_{so}^2 + P_{x0}}{2m_0c^2}} = \frac{\gamma_0 + \sqrt{\gamma_0^2 - 1}}{2m_0c}, \quad (8) \]

\( \gamma_0 \) is the Lorentz factor of an electron before the interaction, and for sufficiently high energies

\[ B \approx \frac{\gamma_0}{m_0c}. \quad (8a) \]

Here \( \eta = \omega \tau_f - kx_f \) is the spatio-temporal interval between the electron coordinate \( x_f \) and the initial point of interaction (mirror), \( \tau_f \) is the interaction duration, and \( \phi_0 \) is the initial phase of some chosen electron when it comes to interact with the electromagnetic field. Note that these results coincide with the formulas derived in [4], where the problem solution was carried out by the Hamilton–Jacobi method for the case when an electron is at rest in the mean. For all electrons in the bunch, which does not have transverse sizes (linear bunch), the interval \( \eta = \omega \tau_f - kx_f \) is constant and is determined by the coordinate \( x_f \) of the observation point. If we choose

\[ \eta = \omega \tau_f - kx_f = 2\pi n, \quad (9) \]

then at the observation point \( x_f(\tau_f) \) all electrons of the bunch will have the same values of the momentum, which they had before the interaction. In the following, we choose \( n = 1 \), i.e., \( \eta = \omega \tau_f - kx_f = 2\pi \), from which one can determine the particle residence time in the interaction region, \( \tau_f = 2\pi/\omega + kx_f/\omega \). In the case, when \( \eta = 2\pi \), from Eq. (4) we obtain

\[ x_f = \frac{2B}{k} \left[ p_{x0} + \frac{Be^2E_0^2}{2\omega^2} \left( 1 + 2\sin^2 \phi_0 \right) \right] 2\pi \quad (10) \]

and, hence,

\[ \tau_f = \frac{2\pi}{\omega} \left[ 1 + 2B \left( p_{x0} + \frac{Be^2E_0^2}{2\omega^2} \left( 1 + 2\sin^2 \phi_0 \right) \right) \right]. \quad (10a) \]

Expression (10a) can be represented as

\[ \tau_f = \tau_0 + \tau_1 \sin^2 \phi_0, \quad (10b) \]

where

\[ \tau_0 = \frac{2\pi}{\omega} \left[ 1 + 2B \left( p_{x0} + \frac{B^2e^2E_0^2}{2\omega^2} \right) \right] \quad \text{and} \quad \tau_1 = \frac{4\pi B^2e^2E_0^2}{\omega^3}. \quad (10c) \]

Let us denote the coordinate of some particle in the bunch by \( u \). After the interaction with the wave this coordinate acquires the increment \( [\tau(u) - \tau(u + du)]v_{so} \) and takes the value \( dv = du + [\tau(u) - \tau(u + du)]v_{so} \). Let the charge distribution in the bunch before its interaction with the laser field be arbitrary and be given by the expression \( \eta(u) = dN/du \), while after the interaction, i.e., at the point \( x = x_f \), it is equal to \( \eta(v) = dN/\omega \). The origin of coordinates in the bunch can be chosen so that the field phase \( \phi_0 = 0 \) corresponds to the electron with the coordinate \( u = 0 \), i.e., the electron with the initial coordinate \( u = 0 \) interacts with the maximum of the electromagnetic field, and \( v \) is the electron coordinate in the bunch after the interaction. Naturally, in this case it is convenient to define the initial phase \( \phi_0 \) as \( \phi_0(u) = \omega t(u) = u\omega/v_{so} = u\omega/\omega = ku/\beta_0 \), where