INTRODUCTION

The problem of rolling of rigid cylinder over the boundary of plastically deformable semispace is challenging for the theory of rolling friction and the technology of surface plastic deformation. Yu.A. Ishlinskii treated the case of rolling a rigid cylinder over a viscoelastic and perfectly elastic semispace taking into account sliding at the interface [1, 2]. The force of resistance induced by rolling in model [1] is due to the distribution of asymmetric pressure on the cylinder in response to the viscosity of the material. The force of resistance induced by rolling in the model [2] is due to the sliding in the contact area and friction according to the Amontons–Coulomb law. The sliding of the indenter and the rolling of the cylinder over a fine viscoelastic layer at the elastic semispace boundary under loads is weak to cause plastic deformation are considered in [3] in respect to the rolling friction problem.

The plastic deformation of the surface layer appears to considerably exceed the elastic deformation under heavy loading during extreme friction and wear. The stationary and unstationary processes of plastic surface layer deformation are used in machine-building technology to promote the wear resistance and fatigue strength of pieces [4–6]. The plastic deformation, residual stresses, plastic layer thickness, the surface cleanness, the wear resistance, and fatigue strength of pieces depend on the shape of the indenter, force parameters of the process, lubrication, and coatings applied to the tools [7, 8].

The plastic deformation of the surface can be analyzed using the perfectly plastic body theory [10]. A stationary plastic region was studied in [11] when a smooth cylinder was rolled over a perfectly plastic semispace boundary [11] with the small parameter method. The approximate analysis of cylinder rolling over the perfectly plastic semispace boundary is analyzed and the rolling friction papers are reviewed in [12]. The calculations of cylinder rolling without sliding over the plastic semispace boundary are known when the finite element method is used [13]. However, severe deformation in the plastic contact area, the problem of the unknown boundaries of the plastic area, the singularity of fields of stresses, and the velocities around the contact between the free plastic boundary and the cylinder hinder the elastoplastic modeling of the rolling and sliding of the cylinder.

The present paper deals with the stationary plastic deformation of the surface layer during the rigid cylinder rolling and sliding over the semispace boundary. The limiting forces and moments act on the cylinder and, without them, stationary plastic flow is impossible. The force and deformation parameters of the plastic deformation of the surface are obtained by the numerical solution of problems indicated in [14–16].

PRINCIPLES OF THE THEORY

When a rigid cylinder with radius \( R \) rolls over the boundary of a perfectly plastic semispace, a stationary plastic area appears that depends on forces \( F \) and \( Q \) and moment \( M \) (figure). Let us introduce the dimensionless variables by assuming the velocity of the cylinder axis \( V \), the contact arc \( R_0 \), and the doubled yield stress at shear \( 2k \) as the unity of velocity, length, and stress, respectively. The plastic region is motionless in coordinates \( \{x, y\} \) that relate to the cylinder lower
point, while the semispace moves with the singular velocity \( V = 1 \). The velocities are continuous along the boundary of the plastic area with the semispace. During the stationary plastic flow the boundaries \( AB \) and \( OA \) are the current lines. The cylinder rotates around the axis with the angular velocity determined by the ratio \( \omega R = 1 \) when it is rolling without sliding at point \( O \). In this case, the sliding velocity \( V_c \) of the material points relative to the cylinder is directed from point \( O \) to point \( A \). The friction stress by Prandtl \( \tau_c \) at the contact boundary applied to the cylinder is aimed from point \( A \) to point \( O \), and the moment is \( M > 0 \). If \( \omega R < 1 \) and the value \( \omega \) is small to the extent that \( V_c \) and \( \tau_c \) are aimed from point \( A \) to point \( O \), then \( \omega \) is \( M < 0 \). For a smooth cylinder, the values are \( \tau_c = 0 \) and \( M = 0 \). In this case, the force of resistance to rolling \( F \) is due to the asymmetry of the plastic region. There are three likely cases of the problem in question.

1. smooth cylinder rolling and sliding at \( \tau_c = 0 \), \( M = 0 \), \( 0 \leq \omega R \leq 1 \);
2. cylinder rolling with forward sliding at \( \tau_c > 0 \), \( M > 0 \), \( \omega R = 1 \);
3. cylinder rolling with backward sliding at \( \tau_c < 0 \), \( M < 0 \), \( 0 \leq \omega R < V_c \).

The plastic region and the velocity of plastic flow of the perfectly plastic body are determined by differential equations of Henke and Heinriger along sliding lines as follows:

\[
d\sigma - d\varphi = 0 \text{ along } \xi, \quad d\sigma + d\varphi = 0 \text{ along } \eta,
\]

\[
dV_\xi - V_\eta d\varphi = 0 \text{ along } \xi, \quad dV_\eta + V_\xi d\varphi = 0 \text{ along } \eta,
\]

where \( \sigma \) is the mean stress, \( \varphi \) is the angle of slope of the tangent to the sliding line \( \xi \) with the axis \( x \), and \( V_\xi \) and \( V_\eta \) are the projection of the velocity vector on \( \xi \) and \( \eta \). The velocities are constant along rigid plastic boundaries \( O-B \).

The sliding lines intersect the contact boundary \( OA \) at an angle \( \theta \) determined as follows by contact friction stresses \( \tau_c \) according to Prandtl:

\[
\theta = \frac{\arccos 2\tau_c}{2}, \quad 0 \leq \tau_c \leq 1/2.
\]

The boundary conditions (2.3) and (2.4) determine the angle \( \psi \) of the centered fan of sliding lines at particular point \( A \) during transition from the boundaries \( AB \) to the contact boundary \( OA \). When the cylinder rolls with forward sliding, the angle \( \theta \) is formed between the sliding line \( \xi \) and the tangent to \( OA \). During backward sliding, the angle \( \theta \) is formed between the sliding line \( \eta \) and the tangent to \( OA \). Therefore, the angle \( \varphi \) depends on the direction of material sliding relative to the boundaries of contact with the cylinder.

The mean stress at the point \( O \) is determined from the Henke equation (2.1) for \( \eta \) of the sliding line \( O-B \):

\[
\sigma_0 = -\frac{1 + \pi}{2}
\]

for a smooth cylinder and

\[
\sigma_0 = -\frac{1 + 3\pi/2}{2} + \theta,
\]

\[
\sigma_0 = -\frac{1 + \pi/2}{2} + \theta
\]

for a rough cylinder during forward and backward sliding, respectively.

Formula (2.6) shows that, in forward sliding, the load-bearing capacity of the rigid wedge at point \( O \) is fulfilled if, at this point, \( \tau_c = 0 \) (\( \theta = \pi/4 \)); this is why, in this case, the plastic region shown in the figure can only be plotted if the variable \( \tau_c \) has a value of zero at point \( O \). Linear changes in \( \tau_c \) were assumed in the paper with the maximum value at the point \( A \). The results of solving the problem show an approximately linear dependence between \( \tau_c \) and \( V_c \). This permits one to interpret the variable values \( \tau_c \) as the viscous resistance to the shear of the boundary layer at the interface between the cylinder and the plastic region. In the case of backward sliding, the load-bearing capacity of the rigid wedge at point \( O \) is fulfilled at all values of \( \tau_c \), which we assume is constant at the interface \( OA \).

The condition of stationary of plastic flow (2.3) leads to the relevant problem of calculating the fields of stresses and velocities. In [15, 16], this problem is reduced to solving the nonlinear vector equation of the