Vector Theory of an Induction Motor

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Abstract—Basic concepts for the vector theory of an induction motor are proposed. The vector method is used to derive new systems of equations and limits for torques, phase angles and absolute values of currents, and the flux linkage for nonlinear saturation. The theory, which is confirmed by the experiment, may be used for the system optimization of induction motors and asynchronous electric drives based on the energy performance criteria.

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The proposed theory, which is a development and advancement of [1, 2], is intended for to improve the power and dynamic characteristics of an induction motor fed from a frequency converter with a variable AC phase, frequency, and amplitude. It may be used for synthesizing and optimizing systems for the vector control of electric drives, for analyzing the regimes of induction motors, and for optimizing designs of special-control motors.

Although the theory of the induction motor has been developed for a long time, the motor’s specific performance for the main relations, including torque–current, power–mass, torque–mass, and acceleration–current are still inferior to modern synchronous motors based on rear-earth motors and innovative induction motors. In the case of a controllable induction motor, for large values of the torque variation $D_M = \frac{M_{\text{max}}}{M_{\text{min}}} > 2–4$, these disadvantages may be removed by optimizing vector control with linked variation in the function of the torque $M(t)$ of flux linkage $\Psi_{\text{mopt}}(M, t)$, $\Psi_{\text{rep}}(M, t)$, and the angles of the mutual phase displacements of electromagnetic vectors $\epsilon(M)$ and $\phi(M)$ [1, 2].

To ensure the competitiveness of induction motors with vector control, which are often advantageous in regards to their cost and reliability, it is necessary to improve the power and dynamic performances of an induction motor to limiting values that are physically inherent to its operation principle by introducing new control processes.

The development of a methodology for attaining the dynamic limits of drives is an important task for critical applications [3].

A known advantage of vector control consists of implementing conditions of the vector orientation of the controllable vector $\bar{U}_s$ or $\bar{I}_s$ under which the controllable rotation frequency $\omega_{opt}(t)$ of the $x, y$ coordinate system becomes equal to the actual frequency $\omega(t)$ of the orientation vector of the flux linkage $V_0(t)(\Psi_r, \Psi_m, \Psi_s)$ and its projections are $V_{0x}(t) = |\bar{V}_0(t)|$, and $V_{0y}(t) = 0$. These requirements are met if the phase and frequency of the orientation vector $\varphi_{r, m, s}(t)$ and $\omega_{V0}(t)$ are controlled depending on the dynamic slide $\Delta \omega_{V0}(t)$ determined from the rotor differential equations expressed in coordinates $\bar{V}_0(y)$ [2] as follows:

$$\Delta \omega_{V0}(M^*(t), V_0(t)) = \omega_{V0}(t) - z_p \omega_{M}(t)$$

$$= \omega_{V0}(M(t), V_0(t)) - \omega(t),$$

(1)

where $z_p$ is the number of pole pairs, $\omega_M$ is the mechanical angular velocity of rotation, $M^*(t)$ is the input torque, and $M(t)$ is the torque obtained at the output of the drive.

The problem of attaining the limiting dynamic, power, and specific performance indicators of an induction motor involves a number of optimality criteria, including the following:

1. Maximal ratio of the torque $M$ and the stator amperage $I_s$; i.e., the criterion of the maximal coefficient of electromechanical coupling $K_{em}$ [N m/A];

$$K_{em} = \frac{M}{I_s} \rightarrow \text{max} \text{ for } U_s \leq U_{\text{max}}, 0 < I_s \leq (5–7)I_{\text{sn}};$$

2. Minimal energy losses and total losses of power in the motor,

$$\Delta P_2(M, \omega) = \Delta P_{2\text{min}} \text{ for } \omega_{\text{min}} \leq z_p \omega \leq \omega_{\text{max}}, U_s \leq U_{\text{max}},$$

and $0 < M \leq M_{\text{max}}$;

3. Maximal efficiency $\eta(f_{\text{opt}}, U_s)$ and output mechanical power $P_2$ for the limited permissible losses and heating of the active parts of the motor,

$$\eta(f_{\text{opt}}, \Delta P_{2\text{perm}}) = \eta_{\text{max}}; P_2 = M_{\text{opt}} \omega_{\text{mopt}} \rightarrow \text{max} \text{ for }$$

$$\Delta P_{2\text{min}}(M, \omega_M) = \Delta P_{2\text{perm}}.$$
Since during the synthesis of a high-precision drive the necessary condition consists of an accurate reproduction of the dynamical value of the torque \( M(t) = M^*(t) \) within the broadest possible range \( D_M \), the torque-generating vectors of currents and flux linkage prove to be optimization parameters. Owing to this circumstance, it is necessary to set and solve the following inverse problem of the vector dynamic synthesis: within the ranges of static and dynamic variations of torque, the \( M \) and \( M(t) \) and the frequencies \( \omega_M \) and \( \omega_M(t) \) required for a mechanism, it is necessary to find the laws of optimal vector interactions of currents \( I_s \), \( I_m \), and \( I_r \) and flux linkages \( \Psi_s \), \( \Psi_m \), and \( \Psi_r \), which are compatible under the dynamic and static conditions for the saturation \( \Psi_m(I_m) \) and variation of induction \( L_m(\Psi_m) \).

The proposed vector theory of an induction motor as applied to solving such problems contains six sections, each requiring further development.

1. **Differential equations of electromagnetic processes for saturation.**

The vector relations based on criteria 1–3 are synthesized by analytically solving differential equations for two axes \( x \) and \( y \) with a variable inductance \( L_m(\Psi_m(t)) \) when a law is introduced into these equations that describes the phase shift angle \( \epsilon_{s\Psi}(M) \) of the vector of the stator current \( I_s \) with respect to the vector \( \nabla_0 \) [1, 2].

Under dynamical conditions, the linked variation of the projections \( I_{sx}(t) \) and \( I_{sy}(t) \) of the controllable current vector \( I_s \) is set by the phase law \( \epsilon_{s\Psi}(M^*(t)) \) as a function of the required torque \( M^*(t) \) variable at the drive’s input within a given range \( D_M \).

Introduction of the phase law \( \epsilon_{s\Psi}(M^*(t)) \) into differential equations set the dynamic links between the dynamic projections of the current \( I_s(t) \) and \( I_e(t) \), variation of the vectors of flux linkages \( \Psi_s(t) \), \( \Psi_m(t) \), and \( \Psi_r(t) \) and their phase shifts, thus determining the drive regimes \( U_s(t) \), \( M(t) \), and \( \omega_M(t) \) and the power and dynamic properties of the motor.

2. **The system of phase equations for an induction motor.**

The list of the main parameters of vector control and optimization includes the phase shift angles of the vectors [1] denoted as follows:

- \( \Phi_{ij} = \Psi_j \Psi_i \), \( \epsilon_{ij} = I_j \Psi_i \), \( \Phi = U \Psi \); and
- \( \gamma_{x,m,r} = U \Psi s_{x,m,r} \).

The relation of the phase angles \( \forall \) between the vectors is determined by the phase equations that follow from the vector diagram presented in [1]:

\[
\Phi_{sm} = \epsilon_{sm} - \epsilon_{ss}; \quad \Phi_{sm} = \Psi_s \Psi_m; \quad \epsilon_{sm} = I_s \Psi_m; \quad \epsilon_{ss} = I_s \Psi_s; \quad (2)
\]

\[
\Phi_{mr} = \epsilon_{sr} - \epsilon_{sm}; \quad \Phi_{mr} = \Psi_m \Psi_r; \quad \epsilon_{sr} = I_s \Psi_r; \quad \epsilon_{sm} = I_s \Psi_m; \quad (3)
\]

\[
\Phi_{ss} = \epsilon_{ss}; \quad \Phi_{ss} = \Psi_s \Psi_s; \quad \epsilon_{ss} = I_s \Psi_s; \quad \epsilon_{ss} = I_s \Psi_s; \quad (4)
\]

\[
\Phi = \epsilon_{us} - \epsilon_{ss}; \quad \Phi = U \Psi I_s; \quad \epsilon_{us} = I_s \Psi_s; \quad (5)
\]

\[
\epsilon_{si} = \epsilon_{sm} + \epsilon_{mr} + \epsilon_{rr}; \quad \epsilon_{si} = I_s \Psi_i; \quad \epsilon_{si} = I_s \Psi_i; \quad (6)
\]

\[
\forall_{ij} = \arctan \left( \frac{V_{ij}}{I_{ij}} \right); \quad \forall_j \forall_{ij} = 0; \quad \forall_j \forall_{ij} = \frac{\pi}{2}. \quad (7)
\]

3. **System of frequency equations for electromagnetic vectors.**

In the coordinate system of the vector of stator flux linkage \( \Psi_s \), the condition of vector orientation is observed \( \Psi_{sx}(t) = \Psi_s(t) \), \( \Psi_{sy} = 0 \), and \( \omega_s(t) = \omega_{sx}(t) \) and the dynamics is described by the simplest vector equations [2]:

\[
I_{sx}(t) = I_s(t) \cos \epsilon_{ss}(t) = \frac{1}{R_s} \left( U_{sx}(t) - \frac{d\Psi_s(t)}{dt} \right); \quad (8)
\]

\[
I_{sy}(t) = I_s(t) \sin \epsilon_{ss}(t) = \frac{1}{R_s} \left( U_{sy}(t) - \omega_{sx}(t) \Psi_s(t) \right); \quad (9)
\]

\[
M(t) = \frac{3}{2} \epsilon \Psi_s(t) I_s(t) \sin \epsilon_{ss}(t); \quad (10)
\]

When the flux linkage \( \Psi_s(t) \) changes, the amplitude of the phase voltage is equal to

\[
U_s(t) = \sqrt{\left[ R_s I_s(t) \cos \epsilon_{ss}(t) + \frac{d\Psi_s(t)}{dt} \right]^2 + \left[ R_s \frac{2M(t)}{3(\epsilon - 2)} \Psi_s(t) + \omega_{sx}(t) \Psi_s(t) \right]^2}; \quad (11)
\]

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