Investigation of Turbulence Influence on the Particle Resistance in a Two-Phase Flow

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Abstract—The results of studying the resistance of a regular spherical particle lattice by means of the ANSYS/FLOTRAN software are presented; the lattice is streamlined by a turbulent incompressible flow at different values of the Re number and turbulence parameters. It is shown that the interference of neighboring particles and their resistance coefficients is unsymmetrical. A phenomenon of a significant influence of local turbulence parameters (scale and intensity) on the single particle resistance is described.

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Formulas of Stocks (1), Glushko (2) and other relations that are usually used in determining the resistance coefficients of particles in a two-phase medium are true for a single sphere being streamlined by an incompressible flow in an unbounded space [1]:

\[ C_x = \frac{24}{Re} ; \] (1)

\[ C_x = \frac{21.12}{Re} + 6.3/\sqrt{Re} + 0.25 . \] (2)

In the two-phase medium motion we have a set of a large number of condensed phase (K-phase) particles distributed in some way over the gas volume. At large values of the C-phase fraction of total mass that are characterized of the solid propellant rocket engine and the air-breathing rocket engine (z \approx 0.3 and more), the particles can exert an interference on the resistance coefficients of the neighboring particles. Let us calculate the resistance coefficients of the C-phase particles with regard for this factor and take as the assumptions that:

--- all particles are of the same spherical shape with the \( r_p \) radius;

--- particles are distributed uniformly in the gas volume.

By an uniform particle distribution in a volume is meant that the particle centers are fixed in the lattice nodes having a three-dimensional symmetry. There are three kinds of such lattices: tetrahedral, cubic and prismoidal. In the first case, an elementary lattice cell is a regular tetrahedron, in the second case, it is a cube and in the third case, it is a regular trihedral prism with a height equal to a triangle side.

With the uniform particle distribution each spherical particle of the \( V_p \) volume is inside the elementary gas cell of \( V \) in volume. The ratio of the volumes \( V/V_p \) can be found if the expression for the C-phase fraction of total mass \( z \) is written:

\[ V/V_p = 1 + \rho, (1-z)/(\rho z) , \] (3)

where \( \rho \) is the gas density; \( \rho_c \) is the C-phase density.

Let us consider the prismoidal distribution of particles in the two-phase medium. The prismoidal lattice has no complete three-dimensional symmetry but it is well adapted to the cylindrical volume. In this case the elementary gas cell corresponding to each particle is of a regular hexahedral prism shape with a height equal to the inscribed cylinder diameter. In the cylindrical volume that is occupied by the
Two-phase medium all particles will be built in chains in the cylinder axis direction, and each chain is enclosed in its hexahedral gas tube.

In calculating the resistance of the prismoidal sphere lattice, it is assumed that the vector of the undistributed gas velocity coincides with the direction of the hexahedral gas tube axes. In this case in virtue of three-dimensional symmetry the tube faces will be impermeable for gas, and the problem of calculating the sphere lattice resistance is reduced to the calculation of the chain resistance of spheres enclosed into the hexahedral tube with the impermeable walls.

To simplify the solution of a three-dimensional problem by reducing it to the axisymmetric case, we will replace the hexahedral tube by a cylindrical one having a radius equal to the radius of a circle inscribed into a hexagon. In this case the elementary gas cell corresponding to a particle will be a cylinder with a height equal to the diameter.

The sphere resistance in the cylindrical volume for the incompressible fluid was determined with the use of the ANSYS/FLOTRAN software. In the calculations the following boundary conditions were assumed: in the inlet cylinder section the velocity components \( v_x = u; v_y = 0 \) were specified; in the outlet section—zero pressure, on the axis and on the cylindrical surface—conditions of impermeability (symmetry), on the particle surface—conditions of an adiabatic wall. The force of particle resistance was determined as a difference of the integral average pulses in the inlet and outlet cylinder sections.

A grid with a 100 × 100 node number was used in the calculations. When arranging nodes in the radial direction, their thickening was used so that the cells adjacent to the particle be of about identical size in the circumferential and radial directions. Three ratios of the cylinder and particle radii \( \frac{r_{cyd}}{r_{cy}} = 5; 10 \) and 15 \( (\frac{r_{cyd}}{r_{cyd}} = r_{cy}/r_p) \) were considered. Figure 1 presents the computation grid for the case \( \frac{r_{cyd}}{r_{cy}} = 5 \).

![Fig.1.](image)

The results of the experimental determination of the sphere resistance force in the incompressible fluid within the \( Re = 0.01–1000 \) range are given in [2]. This range completely overlaps the Re range with respect to the relative velocities between the gas and particles in the two-phase sub- and supersonic flows. The experiments were carried out by different authors using two different methods. At the small Re numbers the resistance coefficients were determined by measuring the rate of small metal sphere precipitation under the gravity in a vessel filled with a fluid of large viscosity. At large Re numbers a fluid or gas flow ran on the stationary sphere. The error of the experiment at \( Re \leq 0.5 \) was less than 1%; as the Re number increased, it grew and amounted to 3–4 % at \( Re = 500 \).

Table 1 presents the experimental values of the resistance coefficients of the sphere \( C_x \) at some selective values of the Re number. Shown here are also the resistance coefficients calculated by Glushko’s formula (2) that were obtained by approximation of these experimental data. It is seen that in the range of the Re numbers \( Re = 10–1000 \), the relative error of Glushko’s formula \( \varepsilon \) is about 1 %. If the Re number is reduced less than 10, the error starts growing.

**Table 1.**

<table>
<thead>
<tr>
<th>Re</th>
<th>( C_x ) (experimental)</th>
<th>( C_x ) (Glushko)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>5.0 ± 0.5</td>
<td>4.9</td>
</tr>
<tr>
<td>1.0</td>
<td>5.0 ± 0.5</td>
<td>4.9</td>
</tr>
<tr>
<td>10.0</td>
<td>5.0 ± 0.5</td>
<td>4.9</td>
</tr>
</tbody>
</table>

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