STRUCTURAL MECHANICS AND STRENGTH
OF FLIGHT VEHICLES

Analysis of Chaotic Vibrations for the Distributed Systems
in the Form of the Bernoulli–Euler Beams
Using the Wavelet Transform

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Abstract—A problem on chaotic vibrations of the Bernoulli–Euler beams is formulated. The wavelet transform was first applied to investigate the complex beam vibrations. The validity of results was provided by using two methods of solution, namely, the finite element method and finite difference method.

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The Fourier analysis, that is, the analysis based on the fast Fourier transform, is one of the most spread for investigating the chaotic dynamics of different physical nature. Its application for the distributed mechanical systems such as beams, plates, and shells is discussed in [1–8]. In addition, at present the signal analysis based on the wavelet transform is rapidly progressing. In this paper, we demonstrate the advantage of this line that permits studying the processes of time variation in the vibration character on the basis of a frequency-time spectrum.

PROBLEM STATEMENT

An object of investigation is a single-layer beam (Fig. 1) which is the two-dimensional domain of a space $R^2$ in the Cartesian coordinates. In the indicated coordinates the beam as a two-dimensional domain $\Omega$ is determined by a set of points: $\Omega = \{x \in [0, a]; -h \leq z \leq h\}$. From here on, we will use the following designations: $2h$ is the beam height, $a$ is its length.

The thin flexible beams will be considered [7, 8]. The decisive equations in displacements in accordance with the Euler–Bernoulli model after introduction of the dimensionless parameters by the formulas

$$\bar{w} = w/(2h); \quad \bar{u} = ua/(2h)^\frac{3}{2}; \quad \bar{x} = x/a; \quad \bar{\lambda} = a/(2h); \quad \bar{\theta} = qa^4/(2h)^4 E;$$

$$\bar{T} = t/\tau; \quad \tau = a/c; \quad \bar{c} = \sqrt{Eg/\bar{\theta};} \quad \bar{e}_i = e_i a/c, \quad i = 1, 2$$

have the following form

$$u^{*} + L_1(w) - \ddot{u} - \dot{\varepsilon}_i \ddot{u} = 0;$$

$$1/\lambda^2 \left[ L_2(w) + L_3(u, w) - 1/12 \dot{w}^{IV} \right] - \dot{\varepsilon}_i \ddot{w} + q = 0. \quad (1)$$

Here the bars over the dimensionless parameters are omitted, the derivatives with respect to time $t$ ($0 \leq t < \infty$) are denoted by a point, and the derivatives with respect to the coordinate—by a dash; $L_i(u, w) = u^*w' + u'^*w$, $L_2(w) = (3/2)w^*(w')^3$, $L_3(w) = w^*w'$ are the nonlinear operators; $\varepsilon_1, \varepsilon_2$ are the
dissipation coefficients; \( E \) is Young’s modulus; \( \vartheta \) is the specific weight of the beam material; \( g \) is the free-fall acceleration.

It is necessary to add to system (1) the initial
\[
\begin{align*}
w(x,t)|_{t=0} &= u(x,t)|_{t=0} = 0; \\
\dot{w}(x,t)|_{t=0} &= \dot{u}(x,t)|_{t=0} = 0
\end{align*}
\]  
(2)
and boundary conditions. In this paper, we will consider a case of the hinged immovable fixation of the beam ends:
\[
w(0,t) = w(1,t) = u(0,t) = u(1,t) = w'(0,t) = w'(1,t) = 0.  
\]  
(3)

NUMERICAL ANALYSIS OF CHAOTIC VIBRATIONS

Let the beam is acted upon by the alternating transverse linear load (see Fig. 1):
\[
q = q_0 \sin(\omega_p t),
\]  
(4)
where \( \omega_p \) is the frequency of the external load; \( q_0 \) is its amplitude. To obtain the numerical results, use was made of the following parameters:
\[
\lambda = a/(2h) = 50; \quad \varepsilon_1 = 1; \quad \varepsilon_2 = 0.
\]

The closed systems of nonlinear equations in partial derivatives and boundary conditions are reduced to the ordinary differential equations of the second order with respect to time by using the finite element method (FEM) in the Bubnov–Galerkin form. The ordinary differential equations with respect to time are solved by the Runge–Kutta method of the fourth order of accuracy.

Figure 2 presents the results of investigating two types of vibrations, namely, harmonic \((q_0 = 100)\) and chaotic \((q_0 = 32200)\) for the value of the excitation frequency \( \omega_p = 6.9 \). Here \( w(0.5,t) \) is the variation of the beam deflection value (signal); \( S \) is the spectrum of the signal power (the Fourier spectrum). As is seen, the results obtained with the use of the finite difference method (dotted lines) and the finite element method (dashed lines) are practically coincident.