Acoustic Properties of Globular Photonic Crystals
Based on Synthetic Opals

N. F. Bunkin1*, V. S. Gorelik2**, and V. V. Filatov2

1Wave Research Center, Prokhorov General Physics Institute, Russian Academy of Sciences,
ul. Vavilova 38, Moscow, 119991 Russia
2Lebedev Physical Institute, Russian Academy of Sciences, Leninskii pr. 53, Moscow, 119991 Russia

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Abstract—Acoustic properties of globular photonic crystals based on synthetic opals and composed of closely packed SiO2 globules about 200 nm in diameter are theoretically investigated. Dispersion characteristics of the investigated samples are numerically simulated, and the group velocity of acoustic waves and the effective mass of acoustic phonons are found. It is shown that phononic bandgaps in these photonic crystals are within the gigahertz frequency range. The effective mass of the acoustic phonons corresponding to the edges of the bandgaps is found, and a possibility that bound states of acoustic phonon pairs, biphonons, manifest themselves in the light scattering spectra is discussed.

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1. INTRODUCTION

Modern technologies allow creating new nanomaterials with a periodic structure. Recently, increasing interest has been shown in the so-called photonic crystals (PCs) whose structure is characterized by the refractive index periodically varying in spatial directions [1]. This feature of PCs results in the presence of bandgaps in their vibration spectrum; i.e., waves of particular frequencies do not propagate in the material and are reflected from the PC surface. A noteworthy case is where the length of the wave incident on the PC is at the very edge of the bandgap. Slowing-down of the wave to complete stop should be observed [2]. As applied to electromagnetic waves, this effect was studied, for example, in [2–4]. It seems important to investigate PC properties in the acoustic frequency range as well in order to establish kinematic and dynamic characteristics of phonons in a PC. Synthetic opals composed of amorphous quartz globules several hundred nanometers in diameter were chosen for the investigation. The pores, space between the globules, could be filled with various substances. In this work, the pores of the PC under investigation were filled with air, water, and nanoparticles of gold. The main purpose of the investigation was to find the dispersion relation \( \omega(k) \) of acoustic waves, their group velocity, and the effective mass of acoustic phonons.

*E-mail: nbunkin@kapella.gpi.ru
**E-mail: gorelik@sci.lebedev.ru

2. THEORY OF DISPERSION OF ACOUSTIC WAVES IN A PHOTONIC CRYSTAL

In general, a photonic crystal is a 3D periodic layered medium. Taking into account the specified direction of the wave, we use the approximation of the 3D medium by the effective 1D model (Fig. 1) in this work.

The 1D dielectric medium with two alternating layers can exhibit properties of both a 1D PC and a 3D PC characterized by specified propagation velocities of acoustic waves in each layer. At the first stage, following the optical-acoustic analogy, we consider the electromagnetic wave dispersion law on the basis of the earlier developed theory [4, 5]. To describe PC properties, we use the Kronig–Penney model. In accordance with the method [6], in order to obtain the dispersion relation, we used the plane monochromatic wave approximation with allowance for the boundary conditions at the edges of the layers (see Fig. 1).

Let us consider, as in [6], the simplest periodic layered medium composed of two different substances with the following refractive index profile:

\[
n(z) = \begin{cases} 
n_2, & 0 < z < b, \\
n_1, & b < z < \Lambda.
\end{cases}
\]  

This profile has the obvious property of periodicity

\[
n(z) = n(z + \Lambda).
\]
ACOUSTIC PROPERTIES OF GLOBULAR PHOTONIC CRYSTALS

Here the \( z \) axis is perpendicular to the interface between the layers and \( \Lambda \) is the period. The geometry of this structure is depicted in Fig. 1. To find the Bloch wave corresponding to the electric field vectors, we use the procedure described in [7]. The general solution to the wave equation for the electric field vector can be sought for in the form

\[
E(r, t) = E_0(z) \exp[i(\omega t - k_y y)].
\]

Here it is assumed that the wave propagates in the \((yz)\) plane; \( k_y \) is the wave vector component that remains constant during the propagation through the medium. The electric field within each homogeneous layer can be represented as a sum of the incident and reflected plane waves. Complex amplitudes of these two waves are components of the column vector. Thus, the electric field in layer \( \alpha (\alpha = 1, 2) \) of the \( n \)th unit cell (see Fig. 1) can be written in the form of the column vector

\[
\begin{pmatrix}
a_n^{(\alpha)} \\
b_n^{(\alpha)}
\end{pmatrix}, \quad \alpha = 1, 2.
\]

The distribution of the electric field in the layer under consideration can be represented as

\[
E(y, z) = \begin{pmatrix} a_n^{(\alpha)} \\ b_n^{(\alpha)} \end{pmatrix} \exp[-ik_\alpha z(z - n\Lambda)] + \begin{pmatrix} a_n^{(\alpha)} \\ b_n^{(\alpha)} \end{pmatrix} \exp[ik_\alpha z(z - n\Lambda)] \exp(-ik_y y),
\]

\[
k_\alpha^2 = \sqrt{\left(\frac{n_0 \omega}{c}\right)^2 - k_y^2}, \quad \alpha = 1, 2.
\]

The column vectors are related to each other by the conditions of continuity at the interfaces. As a consequence, only one vector (or two components of different vectors) can be chosen arbitrarily. For TE waves (vector \( E \) is perpendicular to the \( yz \) plane), the condition for the continuity of the components \( E_x \) and \( H_y \ (H_y \sim \partial E_x / \partial z) \) [7] at the interfaces (see Fig. 1) \( z = (n-1)\Lambda \) and \( z = (n-1)\Lambda + b \) leads to the following equations:

\[
\begin{align*}
a_{n-1} + b_{n-1} &= \exp(ik_{2z}\Lambda)c_n + \exp(-ik_{2z}\Lambda)d_n, \\
iki_{\alpha} (a_{n-1} - b_{n-1}) &= ik_{2z} [\exp(ik_{2z}\Lambda)c_n - \exp(-ik_{2z}\Lambda)d_n], \\
\exp(ik_{2z}a) c_n + \exp(-ik_{2z}a) d_n &= \exp(i1_{\alpha} a)n + \exp(-i1_{\alpha} a) b_n, \\
iki_{\alpha} [\exp(ik_{2z}a) c_n + \exp(-ik_{2z}a) d_n] &= ik_{1z} [\exp(i1_{\alpha} a)n - \exp(-i1_{\alpha} a)b_n].
\end{align*}
\]

These four equations can be written as a system of two matrix equations

\[
\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} \exp(ik_{2z}\Lambda) & \exp(-ik_{2z}\Lambda) \\ \exp(ik_{1z}\Lambda) & \exp(-ik_{1z}\Lambda) \end{pmatrix} \begin{pmatrix} c_n \\ d_n \end{pmatrix} = \begin{pmatrix} \exp(i1_{\alpha} a) & \exp(-i1_{\alpha} a) \\ k_{1z}/k_{2z} & -k_{1z}/k_{2z} \end{pmatrix} \begin{pmatrix} a_n^{(\alpha)} \\ b_n^{(\alpha)} \end{pmatrix}.
\]

\[
a_n \equiv a_n^{(1)}, \quad b_n \equiv b_n^{(1)}, \quad c_n \equiv a_n^{(2)}, \quad d_n \equiv b_n^{(2)}.
\]

Eliminating the column vector \((c_n, d_n)^T\) from this system, we obtain the matrix equation

\[
\begin{pmatrix} a_{n-1} \\ b_{n-1} \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} a_n \\ b_n \end{pmatrix}.
\]