Metamaterial Layer in Rectangular Waveguide

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Abstract—The changes in the dispersion characteristics of modes of isotropic metamaterial layer in a rectangular waveguide have been analyzed. It is shown that metal walls significantly affect the mode dispersion due to the redistribution of energy fluxes and broaden the waveguiding properties of the layer to the propagation range of fast waves. A variation in the waveguide parameters makes it possible, in particular, to implement an interesting class of eigenmodes with fast forward and slow backward waves.

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INTRODUCTION

The interest in the electrodynamics of metamaterials with negative components of permittivity (\(\varepsilon\)) and permeability (\(\mu\)) tensors is primarily caused by their application potential in waveguiding channels and electronic devices [1]. For example, such metamaterials can be used to miniaturize waveguides and as delay systems maintaining both forward and backward waves [2, 3]. It is well known that an isotropic layer with \(\varepsilon<0\) and \(\mu<0\) guides forward and backward TE and TM surface modes. Their propagation constants \(h(\mathbf{E}_i, \mathbf{H}_i \sim \exp(\imath \omega t - \imath h z))\) satisfy the dispersion relations [4]

\[
\text{TE: } -\frac{1}{\mu} \tilde{\kappa} \tanh(\tilde{\kappa} d) = \kappa, \quad (1a)
\]

\[
\text{TM: } -\frac{1}{\varepsilon} \tilde{\kappa} \tanh(\tilde{\kappa} d) = \kappa, \quad (1b)
\]

where \(\kappa = \sqrt{h^2 - k_0^2}\) and \(\tilde{\kappa} = \sqrt{h^2 - k_0^2 \varepsilon \mu}\) are the transverse wavenumbers in vacuum and medium, respectively; \(k_0 = \omega/c\); and \(d\) is the layer half-thickness. The minus and plus signs in (1) correspond, respectively, to the modes with symmetric and antisymmetric \(z\) component of the electromagnetic field.

The waveguiding properties of a metamaterial layer were thoroughly investigated in [4–8]. A detailed classification of modes and specific features of dispersion characteristics were reported in [9, 10], and the anisotropy effect was analyzed in [11].

In this study we analyze the changes in the dispersion characteristics of modes of a metamaterial layer placed in a rectangular metal waveguide (Fig. 1).

For definiteness, we will specify (as in [10]) the following frequency \(\omega\) dependences of the macroscopic medium parameters \(\varepsilon(\omega)\) and \(\mu(\omega)\)\(^1\):

\[
\varepsilon = 1 - \frac{\omega_p^2}{\omega^2}, \quad \mu = 1 - \frac{F\omega_\mu^2}{\omega^2 - \omega_\mu^2}. \quad (2)
\]

Here, \(\omega_p\) and \(\omega_\mu\) are the parameters determined by the resonance properties of the metamaterial and \(F\) is the form factor (on the order of unity). In the further numerical calculations \(F\) is assumed to be 0.56.

Let us first consider two auxiliary problems of a layer placed between infinite metal walls oriented parallel (Sec. 1) and perpendicularly (Sec. 2) to the layer–vacuum interfaces, and then analyze the general case of rectangular waveguide (Sec. 3).

1. METAL WALLS ORIENTED PARALLEL TO THE INTERFACES

Let a flat metamaterial layer \((-d < x < d)\) be located between ideal metal planes \(x = \pm a\) (\(a > d\)) (see Fig. 1, \(b \to \infty\)).

In this case, there are TE and TM transverse modes in the layer. However, one cannot apply the duality principle to them because the zero boundary conditions on the metal are imposed on only the tan-

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\(^1\)Such relations are characteristic of metamaterials composed of thin wires and split ring resonators, which are realized in practice [4].
The electric field in TE waves propagating along the z axis is everywhere parallel to the y axis and turns to zero on the metal walls, \( E_y(x = \pm a) = 0 \). Taking into account the continuity of \( E_y \) and magnetic field tangential component \( H_y \) on the layer boundaries \( x = \pm d \), we obtain the dispersion relation

\[
-\frac{1}{\mu} \kappa \tanh^{-1}(\kappa d) = \kappa \frac{1}{\tanh[\kappa(a - d)]} \tag{3}
\]

with the same designations as in formula (1) for a single layer.

It is known [12] that the vacuum–metamaterial interface can direct true surface waves (TSWs), which are exponentially pressed to it. At \( \mu < -1 \) and \( \varepsilon \mu < 1 \) these TE waves are forward, while at \( -1 < \mu < 0 \) and \( \varepsilon \mu > 1 \) they are backward. As was noted above, an isolated layer also maintains symmetric and antisymmetric slow modes with \( h > k_0 = \omega/c \), which pass into TSWs at \( k_0 d \to \infty \) [10].

Four typical situations can be considered for a layer between metal walls oriented parallel to the interfaces. In Fig. 2 they are described by frequency dependences of \( \gamma(\omega) = h(\omega)/k_0 \) for symmetric (Figs. 2(a) and 2(b)) and antisymmetric (Figs. 2(c) and 2(d)) modes, which pass into forward TSWs at \( d/a \to 0 \) (Figs. 2(a) and 2(c)) and backward TSWs at \( k_0 d = (\omega_\mu/c) d \to \infty \) (Figs. 2(b) and 2(d)). The results are shown for five different \( d/a \) values:

\[
\begin{align*}
\left( \frac{d}{a} \right)_1 &= 0.2, & \left( \frac{d}{a} \right)_2 &= 0.3, \\
\left( \frac{d}{a} \right)_3 &= 0.4, & \left( \frac{d}{a} \right)_4 &= 0.5, & \left( \frac{d}{a} \right)_5 &= 0.6.
\end{align*}
\tag{4}
\]

The dispersion relations \( \gamma_{1,2}(\omega) \) for the TSWs at the vacuum–metamaterial interface (\( \gamma_1(\omega) \)) and the corresponding modes of a layer made of the same metamaterial (\( \gamma_2(\omega) \)) with a thickness \( k_0 d = 0.75 \) are shown by bold lines in Fig. 2. Note that in the cases shown in panels (b) and (d), where the layer mode is formed by backward TSWs, the curves \( \gamma_2(\omega) \) always have a region of ambiguity (bend), within which two surface waves (forward and backward) with different delays simultaneously exist at the same frequency. If the layer mode is formed by forward TSWs, the region of ambiguity is formed for only antisymmetric modes and in only sufficiently thin layers (see Fig. 2(c)).

As can be seen in Fig. 2, metal walls differently affect the dispersion characteristics of layer modes, depending on the presence (b, c, d) or absence (a) of region of ambiguity in the curve \( \gamma_2(\omega) \).

Let us first analyze the case shown in Fig. 2(a), where the dependence \( \gamma_2(\omega) \) is unambiguous. At \( a \gg d \) the metal walls weakly affect slow (\( \gamma(\omega) > 1 \)) layer modes. However, there are fast waves with \( \gamma(\omega) < 1 \), transferring a finite energy flux, which correspond to the modes of conventional planar waveguide.

With an increase in the ratio \( d/a \) (decrease in size \( a \)), the frequency range in which modes propagate narrows, and their group velocity decreases. Beginning with some \( a \) values (depending on both the layer thickness and metamaterial properties), a bend appears on the dependence \( \gamma(\omega) \), and not only forward but also backward waves (in which the negative energy flux in the metamaterial exceeds the positive flux in vacuum) can propagate in a certain frequency range. At \( d/a \to 1 \) the point of transition from forward to backward wave shifts to infinity, and forward waves vanish.

The main difference of the second case (see Figs. 2(b)–2(d)) from the first one is pronounced in the other mechanism of forward–wave rejection.

If there was a strongly delayed backward wave in the isolated layer (see Figs. 2(b)–2(d)), with a decrease in the distance \( a \) between metal walls, the point of its transformation into a forward wave shifts to smaller \( \gamma \) values (to zero). As a result, forward waves vanish, as well as the bend in the dispersion curve.

It should be noted that, varying the position of metal walls, one can pass from fast to slow waves at \( \gamma(\omega) = 1 \), thus implementing the most interesting class of eigenmodes with fast forward and slow backward waves.

1.2. TM Modes

For the TM modes of a metamaterial layer between metal walls oriented parallel to the interfaces, the dispersion equation has the form