MODELING IN PHYSICAL AND TECHNICAL RESEARCH

Analytical Synthesis of Functional Observers for Systems with Signal Perturbations

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Abstract—An algorithm for the analytical synthesis of functional observers is proposed which is based on a nondegenerate transformation of a state vector using the technology of matrix canonization and methods for solving linear matrix equations of arbitrary dimension. Conditions of the solvability of the synthesis problem and the invariance of the constructed observer to signal perturbations in the form of a system of linear matrix equations are formulated.

Keywords: functional observer, synthesis algorithm, matrix canonization, signal perturbation, invariance.

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INTRODUCTION

A sufficiently important applied problem that often arises in the study, design, and operation of complex technical systems is the estimation of their basic parameters and state vectors. This may be necessary, for example, in solving problems of control [1, 2] or diagnosis [3] when not all of the state vector components can be directly measured.

The state vector is most often estimated by using special devices (algorithms), which are observers; they allow restoring information from known input and output signals [4–6]. Aside from full-order observers and reduced-order observers, there are also so-called functional observers, which allow estimating not all state variables, but a certain set of linear combinations of them [7]. Such observers can be used, for example, to generate control signals with the use of a modal controller [8] or to form a generalized performance criteria during diagnosis in a real time mode.

The aim of this work is to develop an algorithm for analytical synthesis of functional observers on the basis of a special representation of the mathematical model of the system in a state space and to formulate conditions under which the obtained functional observer becomes invariant to the action of external perturbations.

FORMULATION OF THE PROBLEM

Let us consider a dynamic object which is described by the state-space equations

$$\dot{x} = Ax + Bu + Hf; \quad y = Cx. \quad (1)$$

Here $x \in \mathbb{R}^n$, $u \in \mathbb{R}^s$, $y \in \mathbb{R}^m$, and $f \in \mathbb{R}^k$ are the vectors of state, control, output, and external perturbations; $A, B, C,$ and $H$ are numerical matrices of appropriate dimensions. It is assumed that all of the coefficient matrices are known, the state vector and the vector of external perturbations are not available for direct measurement, the control vector and the output vector can be measured with high accuracy, the pair $(A, C)$ is completely observed after Kalman, all the matrix output rows are linearly independent, and $n \geq \max(m, s, k)$. 

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Suppose that the sought information about the processes occurring in the object under study is contained in the vector \( g \) defined by the relation

\[
g = Kx. \tag{2}
\]

Here \( g \in \mathbb{R}^p \) and \( K \) is a numerical matrix of appropriate dimensions.

It is required to synthesize a dynamic system of order \( p \), which would use the known information about the signals \( y \) and \( u \) to form the vector 

\[
\varepsilon(t) = \hat{g}(t) - g(t) \to 0 \quad \text{at} \quad t \to \infty. \tag{3}
\]

**PRELIMINARY RELATIONS**

We solve this problem by using the matrix canonization technology [9], the essence of which lies in the fact that a matrix \( M \) of size \( m \times n \) is assigned to four matrices \( \tilde{M}_L \), \( \tilde{M}_R \), \( \bar{M}_L \), and \( \bar{M}_R \), which satisfy the equality

\[
\begin{bmatrix}
\tilde{M}_L \\
\bar{M}_L
\end{bmatrix}
\begin{bmatrix}
\tilde{M}_R \\
\bar{M}_R
\end{bmatrix}
= \begin{bmatrix}
I & 0 \\
0 & 0
\end{bmatrix}.
\]

Here \( \tilde{M}_L \) and \( \tilde{M}_R \) are the left and right matrix divisors of unity, and \( \bar{M}_L \) and \( \bar{M}_R \) are the left and right matrix divisors of zero. The product \( \tilde{M}_R \bar{M}_L \) is denoted as \( \hat{M} \) and is called a combined canonizer.

We define the state vector of the dynamic object from the second equation of system (1) by using the methods for solving linear matrix equations of arbitrary form [9]:

\[
x = \hat{C}y + \hat{C}^Rz, \tag{4}
\]

where \( z \) is some unknown vector of length \( n - m \).

Equation (4) is always valid because, according to the assumptions made above, the output matrix has a full row rank, i.e., \( \hat{C}^L = \emptyset \).

Substituting Eq. (4) into Eq. (2), we obtain

\[
g = K\hat{C}y + K\hat{C}^Rz. \tag{5}
\]

We denote \( \mu = K\hat{C}^Rz \). Then, considering this expression as a linear matrix equation, we define the vector \( z \):

\[
z = \bar{C}^R\mu + \bar{C}^R\eta. \tag{6}
\]

Here \( \eta \) is some unknown vector whose dimension is equal to the number of linearly dependent columns of the matrix \( K\hat{C}^R \).

Equation (6) is valid only if the condition \( \bar{C}^R \) is satisfied, i.e., all the rows of the matrix \( K\hat{C}^R \) should be linearly independent. This implies, in particular, that the number of rows of the matrix \( K \) does not exceed the number of columns of the matrix \( \bar{C}^R \) (\( p \leq n - m \)).

We combine Eqs. (4) and (6) and write the result in matrix form:

\[
x = \begin{pmatrix}
\hat{C} & \hat{C}^R\bar{C}^R & \bar{C}^R\bar{C}^R
\end{pmatrix}
\begin{pmatrix}
y \\
\mu \\
\eta
\end{pmatrix}. \tag{7}
\]

It can be shown that, under the assumptions, the coefficient matrix on the right side of Eq. (7) becomes square nonsingular. To prove this, we transform the matrix as follows:

\[
\begin{pmatrix}
\hat{C} & \hat{C}^R\bar{C}^R & \bar{C}^R\bar{C}^R
\end{pmatrix}
= \begin{pmatrix}
\hat{C} & \hat{C}^R & \bar{C}^R\bar{C}^R
\end{pmatrix}.
\]

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