On $\kappa$ Symmetry Fixing of the D3 Brane in AdS$_5 \times$S$^5$

Hyeonjoon Shin$^*$
School of Physics, Korea Institute for Advanced Study, Seoul 130-722, Korea
(Received 10 June 2015)

In the static gauge, we investigate the supersymmetry realized on a D3 brane in the AdS$_5 \times$S$^5$ background. For the $\kappa$ symmetry fixing condition, the covariant and the Killing spinor gauges are taken. The worldvolume supersymmetry transformation rule for each gauge is then obtained. We show that neither of the two fixing conditions is not suitable for the correct description of the 'vacuum' configuration of the worldvolume theory because both make the 'vacuum' non-supersymmetric as opposed to the usual expectation. Thus, we manifest what the problem of $\kappa$ symmetry fixing is in the study of the D3 brane in the AdS$_5 \times$S$^5$ background.

PACS numbers: 11.25.Uv, 11.25.Tq, 11.30.Pb
Keywords: D-brane, AdS/CFT correspondence, Supersymmetry, Kappa symmetry
DOI: 10.3938/jkps.67.433

I. INTRODUCTION

Among many kinds of D branes, the D3 brane may be a quite attractive object because it gives the familiar four-dimensional gauge theory on its worldvolume at a low energy scale and provides an important seed for the AdS/CFT correspondence [1]. As is well known, the D3 brane, like other D branes, is described by the Dirac-Born-Infeld (DBI) action with the Wess-Zumino (WZ) term in a given background that is the solution of Type IIB supergravity [2–4]. Important ingredients of the action are the spacetime supersymmetry and the local fermionic $\kappa$ symmetry.

If the background is especially the AdS$_5 \times$S$^5$ background, which leads to the first example of the AdS/CFT correspondence, the D3 brane action is efficiently constructed by exploiting the underlying symmetry superalgebra of the AdS$_5 \times$S$^5$ background [5].$^1$ The action is complete and allows the properties of D3 brane itself, for example, like the SL(2,Z)-duality, to be investigated [8, 9]. On the other hand, the action and its symmetries are essential in studying the supersymmetric configurations of the D3 brane in the probe approximation.

In relation to the AdS/CFT correspondence, the most elementary, but important, D3 brane configuration among many possibilities may be the one in which the D3 brane spans the four-dimensional subspace of AdS$_5$ transverse to the AdS$_5$ radial direction. The DBI-type D3 brane action for such a configuration is believed to coincide with the leading infra-red part of the quantum effective action for the $U(1)$ $\mathcal{N} = 4$ vector multiplet obtained by integrating out the massive fields on the Coulomb branch of the $\mathcal{N} = 4$ super Yang-Mills (SYM) theory [10]. Recently, a stronger argument has been given in Ref. [11] that the D3 brane action is the exact effective action, called a highly effective action, for $\mathcal{N} = 4$ SYM theory on the Coulomb branch.

The D3 brane configuration mentioned just above is realized by taking the static or physical gauge for the worldvolume reparametrization symmetry. In exploring the relation with the SYM theory further, two of the basic tasks is to get the supersymmetry transformation rules realized on the D3 brane worldvolume and to investigate the number of supersymmetries preserved by the configuration, which are our interests in this paper. As usual, an important step is to fix the $\kappa$ symmetry properly. We will consider two $\kappa$ symmetry fixing conditions, known as the covariant gauge and the Killing spinor gauge, and investigate the supersymmetry of the D3 brane in static gauge. However, as we shall see, the result is disappointing. For both two $\kappa$ symmetry fixing conditions, the D3 brane configuration under consideration breaks all the spacetime supersymmetries. Obviously, this is far from the usual expectation because the analysis of the open string picture has shown that half of spacetime supersymmetries are preserved. The origin of the trouble is in the $\kappa$ symmetry fixing. Actually, some reports in the literature [5,12] have argued, but without any detailed analysis, that no known $\kappa$ symmetry fixing condition is suitable. The present work is intended to manifest what the problem about $\kappa$ symmetry fixing is.

The organization of this paper is as follows: In the next section, the D3 brane action in the AdS$_5 \times$S$^5$ back-

$^*$E-mail: hyeonjoon@kias.re.kr
$^1$ The usefulness of the symmetry superalgebra has been realized in the construction of the Type IIB superstring action in the AdS$_5 \times$S$^5$ background [6,7].
ground is reviewed briefly. In Sec. III., the worldvolume supersymmetry transformation rules of the D3 brane in the static gauge are derived for two $\kappa$ symmetry fixing conditions, the covariant and the Killing spinor gauges. In Sec. IV., we investigate the supersymmetry preserved by the ‘vacuum’ configuration, that is, the D3 brane without any worldvolume excitation. The discussion follows in the final section. Appendix A gives the explicit expressions for the superfields describing the $\text{AdS}_5 \times \text{S}^5$ background with our notation and convention.

II. D3 BRANE IN AN $\text{AdS}_5 \times \text{S}^5$ BACKGROUND

In this section, we briefly review the action of the spacetime supersymmetric D3 brane in an $\text{AdS}_5 \times \text{S}^5$ background constructed by using the super coset method [5] for the purposes of self-containedness and setting up our notation and convention. The action is composed of two parts, the DBI and the WZ parts:

$$S = S_{\text{DBI}} + S_{\text{WZ}}.$$  

(1)

The expressions of these two parts are given by

$$S_{\text{DBI}} = -\int_{M_4} d^4\sigma \sqrt{-\det(G_{ij} + F_{ij})},$$

$$S_{\text{WZ}} = \int_{M_5} H_5,$$  

(2)

where $M_4$ represents the D3 brane worldvolume and $M_5$ is a five-dimensional manifold with the boundary identified with $M_4$, that is, $\partial M_5 = M_4$.

In the DBI part, $G_{ij}$ is the pullback of the $\text{AdS}_5 \times \text{S}^5$ supergeometry described by the Cartan one-form vectorial superfield $L^a$ onto the worldvolume,²

$$G_{ij} = L^a_i L^b_j \eta_{ab}, \quad L^a_i = \partial_i Z^M L^a_M,$$  

(3)

where $i, j$ are the worldvolume indices ($i, j = 0, 1, 2, 3$), and $F_{ij}$ is a combination of the field strength $F_{ij}$ of the worldvolume gauge field $A_i (F_{ij} = \partial_i A_j - \partial_j A_i)$ and the pulled-back background NS-NS two-form superfield $B$. In the form notation, $F$ is given by

$$F = F - B = dA + 2i \int_0^1 ds L^a_s \wedge \Theta^I \Gamma_a \tau^I_L L^L_s,$$  

(4)

where $L^I$ is the Cartan one-form spinorial superfield and the subscript $s$ in the superfields means that the fermionic coordinate $\Theta$ inside the superfields is replaced by $\Theta \rightarrow s\Theta$. As for the WZ part, we have the supersymmetric closed five-form $H_5$, which is

$$H_5 = -\frac{i}{6} L^a \wedge L^b \wedge L^c \wedge L^d \wedge \Gamma_{abc} \tau^2_L L^L_j,$$

$$= -i F \wedge L^a \wedge L^b \wedge \Gamma_{ab} \tau^2_L L^L_j + \frac{1}{30} \left( \epsilon_1 \ldots \epsilon_5 L^{a_1} \wedge \ldots \wedge L^{a_5} + \epsilon_1 \ldots \epsilon_5 L^{a_1} \wedge \ldots \wedge L^{a_5} \right),$$  

(5)

where the terms enclosed in parentheses imply the presence of a self-dual Ramond-Ramond five-form field strength in the $\text{AdS}_5 \times \text{S}^5$ background.

The D3 brane action in Eq. (1) has at least three symmetries. Firstly, although it is a non-gravitational one, it is invariant under the worldvolume reparametrization $\sigma^i \rightarrow \sigma^i - \lambda^i (\sigma)$,  

(6)

where $\lambda^i (\sigma)$ is the local reparametrization parameter. Secondly, the action is spacetime supersymmetric under the transformations

$$\delta_{\eta} Z^M L^a_M = 2i\eta^I \Gamma^a \Theta^I, \quad \delta_{\eta} Z^M L^I_M = \eta^I,$$  

(7)

where $\eta^I$ is generically the spacetime-dependent transformation parameter. More precisely, the DBI and the WZ parts of the action are supersymmetric separately. Actually, supersymmetry is natural because the super coset method respects the background supersymmetry construction. The last is the local fermionic $\kappa$ symmetry, which is in some sense the most important one because it guarantees the worldvolume supersymmetry after gauge fixing. The $\kappa$ symmetry transformation rules are given by

$$\delta_{\kappa} x^a = \delta_{\kappa} Z^M L^a_M = 0, \quad \delta_{\kappa} \Theta^I = \delta_{\kappa} Z^M L^I_M = \kappa^I,$$  

(8)

where the transformation parameter $\kappa$ satisfies, for the $\kappa$ symmetric projection $\Gamma$,

$$\Gamma^I J \kappa^J = \kappa^I.$$  

(9)

The $\kappa$ symmetry projection is basically the pullback of various gamma matrix products onto the D3 brane worldvolume with the properties $\Gamma^2 = 1$ and $Tr \Gamma = 0$, and its explicit expression is given by

$$\Gamma = \frac{\epsilon_{i_1 \ldots i_4}}{\sqrt{-\det(G_{ij} + F_{ij})}} \times \left( \frac{1}{4!} \gamma_{i_1 \ldots i_4} \tau_2 + \frac{1}{8} \epsilon_{i_1 i_2} F_{i_3 i_4} \tau_1 + \frac{1}{8} F_{i_1 i_2} F_{i_3 i_4} \tau_2 \right),$$  

(10)

where $\gamma_{i_1 \ldots i_n} = \gamma_{[i_1 \ldots i_n]}$ and $\gamma_i$ is the pullback of $\Gamma_a$, that is, $\gamma_i = L^a_i \Gamma_a$. If we use the representation of $\tau$ matrices given in Eq. (A3), $\Gamma$ is written simply as

$$\Gamma = \begin{pmatrix} 0 & \beta_+ \\ \beta_- & 0 \end{pmatrix}$$  

(11)

² Explicit expressions for the superfields with our notation and convention are summarized in Appendix A.